

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Ordinary Level

**ADDITIONAL MATHEMATICS**

**4037/01**

Paper 1

May/June 2004

**2 hours**

Additional Materials: Answer Booklet/Paper  
Graph paper (3 sheets)  
Mathematical tables

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate Answer Booklet/Paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages and **3** blank pages.



*Mathematical Formulae***1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)! r!}$ .

**2. TRIGONOMETRY***Identities*

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

*Formulae for  $\Delta ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 Given that  $y = \frac{3x-2}{x^2+5}$ , find

(i) an expression for  $\frac{dy}{dx}$ ,

(ii) the  $x$ -coordinates of the stationary points.

[4]

2 Find the  $x$ -coordinates of the three points of intersection of the curve  $y = x^3$  with the line  $y = 5x - 2$ , expressing non-integer values in the form  $a \pm \sqrt{b}$ , where  $a$  and  $b$  are integers. [5]

3 (i) Sketch on the same diagram the graphs of  $y = |2x + 3|$  and  $y = 1 - x$ . [3]

(ii) Find the values of  $x$  for which  $x + |2x + 3| = 1$ . [3]

4 The function  $f$  is defined, for  $0^\circ \leq x \leq 360^\circ$ , by

$$f(x) = a \sin(bx) + c,$$

where  $a$ ,  $b$  and  $c$  are positive integers. Given that the amplitude of  $f$  is 2 and the period of  $f$  is  $120^\circ$ ,

(i) state the value of  $a$  and of  $b$ . [2]

Given further that the minimum value of  $f$  is  $-1$ ,

(ii) state the value of  $c$ , [1]

(iii) sketch the graph of  $f$ . [3]

5 The straight line  $5y + 2x = 1$  meets the curve  $xy + 24 = 0$  at the points  $A$  and  $B$ . Find the length of  $AB$ , correct to one decimal place. [6]

6 The table below shows

the daily production, in kilograms, of two types,  $S_1$  and  $S_2$ , of sweets from a small company,

the percentages of the ingredients  $A$ ,  $B$  and  $C$  required to produce  $S_1$  and  $S_2$ .

	Percentage			Daily production (kg)
	$A$	$B$	$C$	
Type $S_1$	60	30	10	300
Type $S_2$	50	40	10	240

Given that the costs, in dollars per kilogram, of  $A$ ,  $B$  and  $C$  are 4, 6 and 8 respectively, use matrix multiplication to calculate the total cost of daily production. [6]

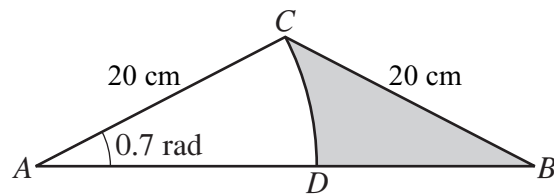
7 To a cyclist travelling due south on a straight horizontal road at  $7 \text{ ms}^{-1}$ , the wind appears to be blowing from the north-east. Given that the wind has a constant speed of  $12 \text{ ms}^{-1}$ , find the direction from which the wind is blowing. [5]

8 A curve has the equation  $y = (ax + 3) \ln x$ , where  $x > 0$  and  $a$  is a positive constant. The normal to the curve at the point where the curve crosses the  $x$ -axis is parallel to the line  $5y + x = 2$ . Find the value of  $a$ . [7]

9 (a) Calculate the term independent of  $x$  in the binomial expansion of  $\left(x - \frac{1}{2x^5}\right)^{18}$ . [3]

(b) In the binomial expansion of  $(1 + kx)^n$ , where  $n \geq 3$  and  $k$  is a constant, the coefficients of  $x^2$  and  $x^3$  are equal. Express  $k$  in terms of  $n$ . [4]

10

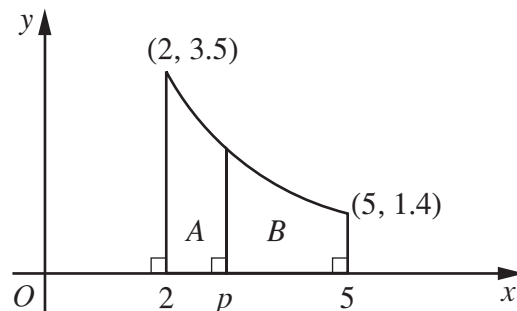


The diagram shows an isosceles triangle  $ABC$  in which  $BC = AC = 20 \text{ cm}$ , and angle  $BAC = 0.7$  radians.  $DC$  is an arc of a circle, centre  $A$ . Find, correct to 1 decimal place,

(i) the area of the shaded region, [4]

(ii) the perimeter of the shaded region. [4]

11



The diagram shows part of a curve, passing through the points  $(2, 3.5)$  and  $(5, 1.4)$ . The gradient of the curve at any point  $(x, y)$  is  $-\frac{a}{x^3}$ , where  $a$  is a positive constant.

(i) Show that  $a = 20$  and obtain the equation of the curve. [5]

The diagram also shows lines perpendicular to the  $x$ -axis at  $x = 2$ ,  $x = p$  and  $x = 5$ . Given that the areas of the regions  $A$  and  $B$  are equal,

(ii) find the value of  $p$ . [5]

12 Answer only **one** of the following two alternatives.

**EITHER**

- (a) An examination paper contains 12 different questions of which 3 are on trigonometry, 4 are on algebra and 5 are on calculus. Candidates are asked to answer 8 questions. Calculate
- (i) the number of different ways in which a candidate can select 8 questions if there is no restriction,
  - (ii) the number of these selections which contain questions on only 2 of the 3 topics, trigonometry, algebra and calculus.

[4]

- (b) A fashion magazine runs a competition, in which 8 photographs of dresses are shown, lettered *A, B, C, D, E, F, G* and *H*. Competitors are asked to submit an arrangement of 5 letters showing their choice of dresses in descending order of merit. The winner is picked at random from those competitors whose arrangement of letters agrees with that chosen by a panel of experts.

- (i) Calculate the number of possible arrangements of 5 letters chosen from the 8.

Calculate the number of these arrangements

- (ii) in which *A* is placed first,
- (iii) which contain *A*.

[6]

**OR**

The table shows experimental values of the variables  $x$  and  $y$  which are related by the equation  $y = Ab^x$ , where  $A$  and  $b$  are constants.

$x$	2	4	6	8	10
$y$	9.8	19.4	37.4	74.0	144.4

- (i) Use the data above in order to draw, on graph paper, the straight line graph of  $\lg y$  against  $x$ , using 1 cm for 1 unit of  $x$  and 10 cm for 1 unit of  $\lg y$ . [2]
- (ii) Use your graph to estimate the value of  $A$  and of  $b$ . [5]
- (iii) On the same diagram, draw the straight line representing  $y = 2^x$  and hence find the value of  $x$  for which  $Ab^x = 2^x$ . [3]

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