UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/01**

Paper 1

May/June 2006

2 hours

Additional Materials: Answer Booklet/Paper
Graph paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
At the end of the examination, fasten all your work securely together.

This document consists of **5** printed pages and **3** blank pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$\sin^2 A + \cos^2 A = 1.$

$\sec^2 A = 1 + \tan^2 A.$

$\operatorname{cosec}^2 A = 1 + \cot^2 A.$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2}bc \sin A.$$

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1 A curve has the equation $y = (x - 1)(2x - 3)^8$. Find the gradient of the curve at the point where $x = 2$. [4]

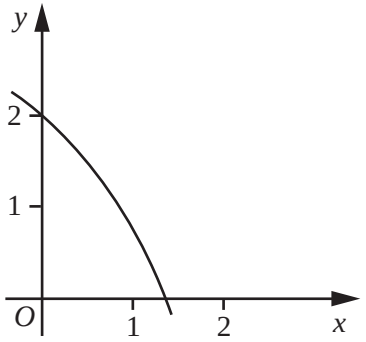
2 The line $y + 4x = 23$ intersects the curve $xy + x = 20$ at two points, A and B. Find the equation of the perpendicular bisector of the line AB. [6]

3 A plane flies due north from A to B, a distance of 1000 km, in a time of 2 hours. During this time a steady wind, with a speed of 150 km h^{-1}, is blowing from the south-east. Find

(i) the speed of the plane in still air, [4]

(ii) the direction in which the plane must be headed. [2]

4



The diagram shows part of the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = p - \mathrm{e}^x$ and p is a constant.
The curve crosses the y-axis at (0, 2).

(i) Find the value of p. [2]

(ii) Find the coordinates of the point where the curve crosses the x-axis. [2]

(iii) Copy the diagram above and on it sketch the graph of $y = \mathrm{f}^{-1}(x)$. [2]

5 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} -2 & -1 \\ 6 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & -1 \\ 4 & 3 \end{pmatrix}$. Find matrices **P** and **Q** such that

(i) $\mathbf{P} = \mathbf{B}^2 - 2\mathbf{A}$, [3]

(ii) $\mathbf{Q} = \mathbf{B}(\mathbf{A}^{-1})$. [4]

6 The cubic polynomial $\mathrm{f}(x)$ is such that the coefficient of x^3 is 1 and the roots of $\mathrm{f}(x) = 0$ are -2, $1 + \sqrt{3}$ and $1 - \sqrt{3}$.

(i) Express $\mathrm{f}(x)$ as a cubic polynomial in x with integer coefficients. [3]

(ii) Find the remainder when $\mathrm{f}(x)$ is divided by $x - 3$. [2]

(iii) Solve the equation $\mathrm{f}(-x) = 0$. [2]

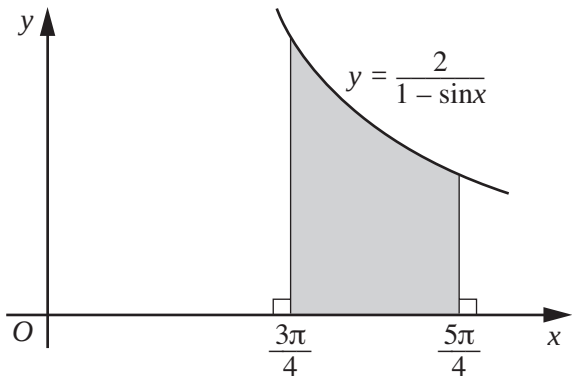
7 A particle moves in a straight line, so that, t s after leaving a fixed point O, its velocity, $v\,\mathrm{m\,s^{-1}}$, is given by

$$v = pt^2 + qt + 4,$$

where p and q are constants. When $t = 1$ the acceleration of the particle is $8\,\mathrm{m\,s^{-2}}$. When $t = 2$ the displacement of the particle from O is 22 m. Find the value of p and of q. [7]

8 **(i)** Given that $y = \dfrac{1 + \sin x}{\cos x}$, show that $\dfrac{dy}{dx} = \dfrac{1}{1 - \sin x}$. [5]

(ii)



The diagram shows part of the curve $y = \dfrac{2}{1 - \sin x}$. Using the result given in part **(i)**, find the area of the shaded region bounded by the curve, the x-axis and the lines $x = \dfrac{3\pi}{4}$ and $x = \dfrac{5\pi}{4}$. [3]

9 **(a)** Given that $u = \log_4 x$, find, in simplest form in terms of u,

(i) x,

(ii) $\log_4\left(\dfrac{16}{x}\right)$,

(iii) $\log_x 8$.

[5]

(b) Solve the equation $(\log_3 y)^2 + \log_3(y^2) = 8$. [4]

10 The function f is defined, for $0° \leqslant x \leqslant 180°$, by

$$\mathrm{f}(x) = 3\cos 4x - 1.$$

(i) Solve the equation $\mathrm{f}(x) = 0$. [3]

(ii) State the amplitude of f. [1]

(iii) State the period of f. [1]

(iv) State the maximum and minimum values of f. [2]

(v) Sketch the graph of $y = \mathrm{f}(x)$. [3]

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11 Answer only **one** of the following two alternatives.

EITHER

The table below shows values of the variables x and y which are related by the equation $y = \dfrac{a}{x + b}$, where a and b are constants.

x	0.1	0.4	1.0	2.0	3.0
y	8.0	6.0	4.0	2.6	1.9

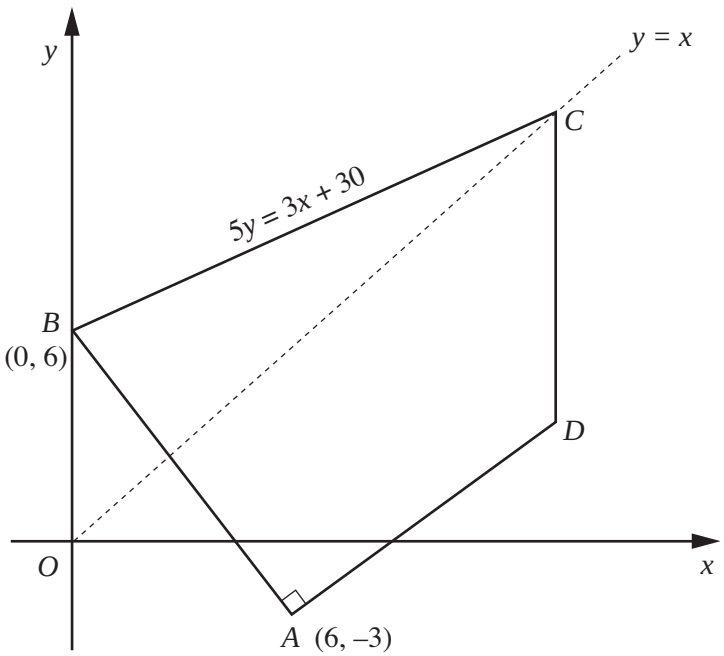
(i) Using graph paper, plot y against xy and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

An alternative method for obtaining a straight line graph for the equation $y = \dfrac{a}{x + b}$ is to plot x on the vertical axis and $\dfrac{1}{y}$ on the horizontal axis.

(iii) Without drawing a second graph, use your values of a and b to estimate the gradient and the intercept on the vertical axis of the graph of x plotted against $\dfrac{1}{y}$. [3]

OR



The diagram, which is not drawn to scale, shows a quadrilateral $ABCD$ in which A is (6, –3), B is (0, 6) and angle BAD is 90°. The equation of the line BC is $5y = 3x + 30$ and C lies on the line $y = x$. The line CD is parallel to the y-axis.

(i) Find the coordinates of C and of D. [6]

(ii) Show that triangle BAD is isosceles and find its area. [4]

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