UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/01**

Paper 1

October/November 2006

2 hours

Additional Materials: Answer Paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

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UNIVERSITY of CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

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1 Express each of the following statements in appropriate set notation.

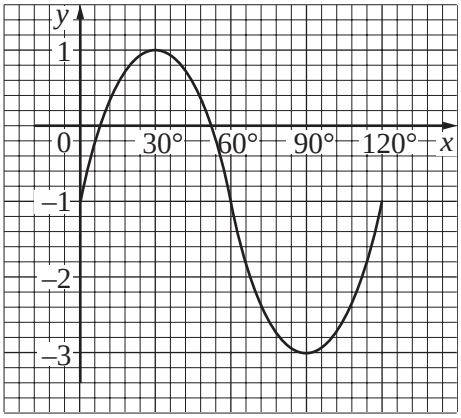
(i) x is not an element of set A.

(ii) The number of elements not in set B is 16.

(iii) Sets C and D have no common element.

[3]

2



The diagram shows part of the graph of $y = a\sin(bx) + c$. State the value of

(i) a,

(ii) b,

(iii) c.

[3]

3 The equation of a curve is $y = \dfrac{8}{(3x-4)^2}$.

(i) Find the gradient of the curve where $x = 2$. [3]

(ii) Find the approximate change in y when x increases from 2 to $2 + p$, where p is small. [2]

4 The vector $\overrightarrow{OP}$ has a magnitude of 10 units and is parallel to the vector $3\mathbf{i} - 4\mathbf{j}$. The vector $\overrightarrow{OQ}$ has a magnitude of 15 units and is parallel to the vector $4\mathbf{i} + 3\mathbf{j}$.

(i) Express $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of **i** and **j**. [3]

(ii) Given that the magnitude of $\overrightarrow{PQ}$ is $\lambda\sqrt{13}$, find the value of λ. [3]

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5 A large airline has a fleet of aircraft consisting of 5 aircraft of type *A*, 8 of type *B*, 4 of type *C* and 10 of type *D*. The aircraft have 3 classes of seat known as Economy, Business and First. The table below shows the number of these seats in each of the 4 types of aircraft.

Type of aircraft \ Class of seat	Economy	Business	First
A	300	60	40
B	150	50	20
C	120	40	0
D	100	0	0

(i) Write down two matrices whose product shows the total number of seats in each class.

(ii) Evaluate this product of matrices.

On a particular day, each aircraft made one flight. 5% of the Economy seats were empty, 10% of the Business seats were empty and 20% of the First seats were empty.

(iii) Write down a matrix whose product with the matrix found in part **(ii)** will give the total number of empty seats on that day.

(iv) Evaluate this total.

[6]

6 Given that the coefficient of x^2 in the expansion of $(k + x)\left(2 - \frac{x}{2}\right)^6$ is 84, find the value of the constant k.

[6]

7 The function f is defined for the domain $-3 \leqslant x \leqslant 3$ by

$$\mathrm{f}(x) = 9\left(x - \frac{1}{3}\right)^2 - 11.$$

(i) Find the range of f. [3]

(ii) State the coordinates and nature of the turning point of

(a) the curve $y = \mathrm{f}(x)$,

(b) the curve $y = \left|\, \mathrm{f}(x) \,\right|$.

[4]

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8 **(a)** Solve the equation $\lg(x + 12) = 1 + \lg(2 - x)$. [3]

(b) Given that $\log_2 p = a$, $\log_8 q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b. [4]

9 A curve has the equation $y = \frac{2x - 4}{x + 3}$.

(i) Obtain an expression for $\frac{dy}{dx}$ and hence explain why the curve has no turning points. [3]

The curve intersects the x-axis at the point P. The tangent to the curve at P meets the y-axis at the point Q.

(ii) Find the area of the triangle POQ, where O is the origin. [5]

10 The cubic polynomial f(x) is such that the coefficient of x^3 is 1 and the roots of f(x) = 0 are 1, k and k^2. It is given that f(x) has a remainder of 7 when divided by $x - 2$.

(i) Show that $k^3 - 2k^2 - 2k - 3 = 0$. [3]

(ii) Hence find a value for k and show that there are no other real values of k which satisfy this equation. [5]

11 **(a)** Solve, for $0° \leqslant x \leqslant 360°$, the equation

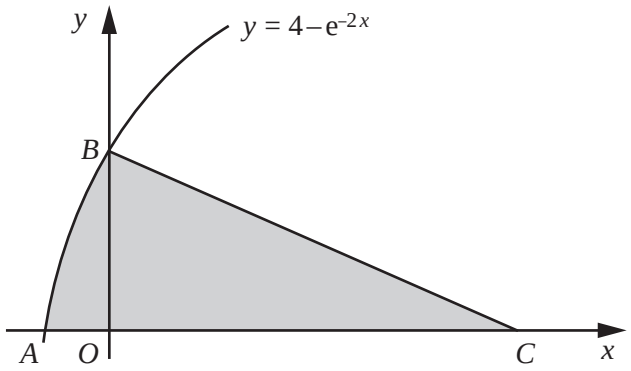
$$2\cot x = 1 + \tan x.$$ [5]

(b) Given that y is measured in radians, find the two smallest positive values of y such that

$$6\sin(2y + 1) + 5 = 0.$$ [5]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows part of the curve $y = 4 - e^{-2x}$ which crosses the axes at A and at B.

(i) Find the coordinates of A and of B. [2]

The normal to the curve at B meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Show that the area of the shaded region is approximately 10.3 square units. [5]

OR

The variables x and y are related by the equation $y = 10^{-A}b^x$, where A and b are constants. The table below shows values of x and y.

x	15	20	25	30	35	40
y	0.15	0.38	0.95	2.32	5.90	14.80

(i) Draw a straight line graph of $\lg y$ against x, using a scale of 2 cm to represent 5 units on the x-axis and 2 cm to represent 0.5 units on the $\lg y$-axis. [2]

(ii) Use your graph to estimate the value of A and of b. [4]

(iii) Estimate the value of x when $y = 10$. [2]

(iv) On the same diagram, draw the line representing $y^5 = 10^{-x}$ and hence find the value of x for which $10^{A - \frac{x}{5}} = b^x$. [3]

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