

# ADDITIONAL MATHEMATICS

Paper 4037/11

Paper 11

## Key message

In order to ensure that candidates receive credit where possible for incorrect answers, they should remember to show all their working. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown.

## General comments

The great majority of candidates were able to attempt most questions, showing a reasonable understanding of the syllabus content and its aims.

With the advent of the new style paper with candidates answering on the actual question paper, both Centres and candidates should be aware of the following:

- Candidates should answer the questions in the correct place on the paper. If they do not have enough space, it is better to do the question on an additional sheet of paper to be attached to question paper booklet and mention the fact that this has been done in the original question space.
- If graph paper is required, it will form part of the question paper booklet. If candidates are asked to sketch a graph, then graph paper is not required.
- It is unnecessary to attach extra booklets of paper to the question paper booklet. If extra paper is needed, then just the sheets used (with candidate name and number included on them) should be attached, rather than several extra blank pages as well.

Candidates are also reminded that they should show all steps of their working, especially in questions where they are requested not to make use of a calculator. Accuracy continues to be a problem, with many candidates losing marks unnecessarily due to premature approximation. Candidates should be encouraged to work to at least 4 significant figures and give their final answer to 3 significant figures unless instructed otherwise. These instructions are on the front of the question paper.

Centres are reminded that, from June 2013, there will be no choice of questions in the papers.

## Comments on specific questions

### Question 1

- (i) Many clearly annotated diagrams were seen. A number of candidates omitted to remove the part of the original line below the x-axis, which was not required, although credit was given if that part was either crossed out or shown as intermittent.
- (ii) A large number of candidates only found the single solution  $x = 4$ .

Answer:  $x = 1, x = 4$

## Question 2

The majority of candidates managed to earn at least some marks on this question. The remainder and factor theorems were generally well understood and any loss of marks was usually down to either poor arithmetic or poor algebraic manipulation. Candidates who attempted to use the division method were generally unsuccessful in getting a remainder in terms of  $a$  and  $b$  only.

Answer:  $a = 5, b = -13$

## Question 3

Most candidates attempted to equate the line and the curve in order to obtain a quadratic equation in  $x$ . Those who attempted to eliminate  $x$  instead of  $y$  were rarely successful. The need to use the discriminant was widely understood but a number of candidates either ignored the negative root or gave the answer as  $-4 < k < 4$ .

Answer:  $k > 4, k < -4$

## Question 4

- (a) This question was generally well answered throughout. The most common error was to interpret “arrangements” as combinations rather than permutations.
- (b) This was also well done, although a common error in part (ii) was to add, rather than multiply, the two terms.

Answer: (a) (i) 15120 (ii) 210 (b) (i) 15504 (ii) 3696 (iii) 56

## Question 5

- (i) The majority of candidates were able to obtain full marks on this question. Differentiation and the  $y$ -intercept were usually correct. A few candidates chose to find the equation of the normal rather than the tangent.
- (ii) Many candidates were able to obtain a cubic from the two equations and to solve it correctly. Those who got part (i) wrong were given credit for attempting to solve their resulting cubic equation.

Answer: (i)  $y = -3x + 4$  (ii)  $(-2, 10)$

## Question 6

- (i) Many different methods of solution to this problem were offered, some succinct and others rather more convoluted, but this question was a good source of marks for most candidates. A number of weaker candidates divided only the left-hand side by  $\cos^2 \theta$  leading to an incorrect answer.
- (ii) Many candidates did not realise that they were given the approach to the solution in part (i) and started again. The negative square root was often not given, resulting in only one solution. Answers were often given in degrees rather than in radians. It should be noted that, although answers in degrees are to be given to one decimal place, answers in radians should be given to three significant figures.

Answer: (ii) 0.902, 2.24

### Question 7

- (i) Only a minority of candidates knew to plot  $\frac{y}{x}$  against  $x$  in order to obtain a straight line. Those who did know invariably gained full marks on the whole of this question.
- (ii) As the question stated “Use your graph”, those candidates who failed to produce a suitable graph in part (i) and substituted values of  $x$  and  $y$  in the original equation could gain no credit.

Answer: (ii)  $A = 3, B = -0.5$

### Question 8

- (a) This question was generally well done and  $\log x^2$  and  $\log 10$  appeared somewhere in most candidates' solutions. Some candidates were, however, unable to use  $\log A - \log B = \log A/B$  correctly, and only a few realised that the solution  $x = -10$  should have been rejected.
- (b) Most candidates recognised the need to change the base of one of the logarithms, although many did not recognise the difference between  $(\log y)^2$  and  $\log y^2$ . Those who reached the stage at which they were to take the square root often missed out the negative root which was also valid.

Answer: (a)  $x = 60$  (b) 25, 0.04

### Question 9

Many candidates were able to find the three correct binomial terms needed to answer this question, although  $(px)^2 = px^2$  was a common error. Those who simplified their equations to  $pq = 1$  and  $4q = 9p$  usually got to the correct answers, whereas those who did not cancel down their original equations often lost their way in convoluted attempts to solve.

Answer:  $p = \frac{2}{3}, q = \frac{3}{2}$

### Question 10

- (i) There were many correct answers to this question, but also a number of candidates who thought that the differential of  $e^{2x}$  was  $e^{2x}$ .
- (ii) Candidates were often unable to produce a quadratic equation from the given information. Of those who did many went on to give the value of  $x$ , rather than  $y$ , as their final answer. As the final answer could be obtained from  $u$ , it was not necessary to find the value of  $x$ , and some of the candidates who realised this also gave the invalid answer of  $y = -2.5$ .
- (iii) Most candidates appreciated what was required to answer this question, but a large number used  $\frac{dy}{dx} = 0.5$  rather than  $-0.5$ .

Answer: (i)  $2e^{2x} - 2e^{-2x}$  (ii) 2.5 (iii)  $-7.25$

### Question 11 Either

This was the less popular choice for candidates, possibly because they did not realise that part (i) was simply a matter of finding the coordinates of the maximum point on the curve.

- (i) Although a few candidates gained full marks in this part, this was often either missed out completely, or composed of irrelevant workings.
- (ii) This proved to be straightforward for those who could integrate accurately.

Answer: (ii) 628

**Question 11 Or**

- (i) Of those who attempted this question, the majority were familiar with the process for finding the equation of a normal to a curve. The most common cause of loss of marks here was incorrect differentiation of  $2 \sin 3x$ .
- (ii) Completely correct solutions to this question were rare. In many cases, either the shaded area, or the area under the curve was taken to be a triangle. Otherwise, incorrect integration of  $2 \sin 3x$  led to loss of marks. Approximation of the equation of the normal as  $y = -\frac{1}{3}x + 1.9$  also led to loss of accuracy marks.

Answer: (i) 1.85      (ii) 0.292

# ADDITIONAL MATHEMATICS

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Paper 4037/12

Paper 12

## Key message

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## General comments

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Centres are reminded that, from June 2013, there will be no choice of questions in the papers.

## Comments on specific questions

### Question 1

- (i) Most candidates were able to make a good start to the paper realising that the answer contained  $(7x - 5)^{\frac{3}{2}}$ . Errors when they did occur involved the numerical factors, with the omission of 7 being the most common. Although omission of the arbitrary constant was not penalised, candidates should be encouraged to include it.
- (ii) Valid attempts at substitution of the limits earned most candidates a method mark. Arithmetic slips were common in the evaluation of the square brackets. The question included the word 'Hence', so a single answer obtained from using a calculator to perform numerical integration failed to score marks as a method using part (i) was not in evidence.

Answer: (i)  $\frac{2}{21}(7x - 5)^{\frac{3}{2}} (+c)$       (ii)  $\frac{74}{21}$

## Question 2

Provided candidates were able to deal with  $2^{2x+2}$  and then go on to form a quadratic equation, many were able to obtain full marks for this question. Problems arose when candidates did not form a quadratic equation. Methods of solving quadratic equations were generally sound. A few candidates lost marks by leaving their answers in terms of  $u$  rather than  $x$ , but most were able to deal competently with either the use of logs or inspection to solve for  $x$ .

Answer:  $-2, 0$

## Question 3

Many completely correct proofs were produced, with most candidates being able to gain some marks if not all. The most common error was to expand out brackets in the denominator of the fraction obtained on the left hand side of the proof and, as a result, not noticing that there was a common factor of  $1 + \cos A$  in both numerator and denominator.

## Question 4

This question was extremely well answered with most candidates coping with the somewhat complicated algebra and usually gaining full marks. There were a few instances of solutions where  $x$  was found but incorrectly labelled  $y$  and vice versa. Solving the resulting quadratic equation was usually done well with only a few candidates making slips in factorisation or use of the quadratic formula.

Answer:  $x = 4, y = -6$  and  $x = \frac{1}{5}, y = \frac{1}{3}$

## Question 5

- (i) Most candidates recognised that they needed to differentiate a product and made a valid attempt to do so. However, many candidates were unable to differentiate  $\ln(3x + 1)$  correctly.
- (ii) Most candidates recognised that they needed to differentiate using the quotient rule but many had problems differentiating  $\tan 2x$  with respect to  $x$ . Factors of 2 were often misplaced or omitted. Those that chose to differentiate using the product rule were usually successful provided that they were able to deal with  $(5x)^{-1}$  or equivalent when differentiating with respect to  $x$ . Errors involving a factor of 5 were most common when this method was used.

Answer: (i)  $(2 - x^2) \frac{3}{3x + 1} - 2x \ln(3x + 1)$       (ii)  $\frac{5x(-2 \sec^2 2x) - 5(4 - \tan 2x)}{25x^2}$

## Question 6

This question was to be done without the use of a calculator. It was therefore important that candidates showed their working as many calculators will produce the required results immediately. The question was set to test the candidates' ability to manipulate surds as required by the syllabus, not their ability to use a calculator for this process.

- (i) Most candidates were able to rationalise correctly and produce the correct result.
- (ii) There were several different approaches that candidates could have made. The most common one was the use of Pythagoras' theorem. This was usually less than successful because candidates were often unable to find the square root of the resulting expression if they had not factorised their expression but expanded it out first. Solutions using trigonometry were usually more successful in gaining candidates full marks, being more straightforward and a lot quicker to produce the given result.

- (iii) Using the given answer from part (ii), most candidates were able to make a correct attempt to find the area of the triangle. However, common errors included using half the base length and also miscopying  $4(\sqrt{3} - 1)$  as  $4\sqrt{3} - 1$ .

Answer: (i)  $4(\sqrt{3} - 1)$  (iii)  $16\sqrt{3} - 24$

### Question 7

- (i) In this type of question, candidates need to ensure that they sketch the graph as requested in the question. Not all candidates showed the points where the curve met the coordinate axes although these were specifically asked for. Too many candidates incorrectly assumed symmetry about the  $y$ -axis and a significantly large number of candidates did not sketch a complete graph, ignoring the domain  $x > 3$  and  $x < -2$  or  $x < 0$ . Candidates were expected to sketch their graph in the space provided on the question paper **not** to plot it on a separate piece of graph paper.
- (ii) Full marks were gained by many candidates, usually dealing correctly with two separate quadratic equations. Many candidates chose to incorrectly discard the solution  $x = -3$ .

Answer: (ii)  $-3, 0, 1, 4$

### Question 8

- (i) Most candidates were able to find the correct arc length with few being unable to deal with radians. Errors usually occurred when finding the length of either  $AX$  or  $BX$ ; the most common error being an incorrect use of trigonometry or use of an incorrect angle. Too many candidates did not work to the appropriate degree of accuracy during the question and answers of 55.5 due to premature approximation were common. Many candidates gave their answer in a completely acceptable exact form.
- (ii) Most candidates were able to find the correct sector area. Similar errors to those in part (i) led to problems with finding the area of  $OAXB$  and premature approximation again. The latter was penalised only once in the entire question.

Answer: (i) 55.6 (ii) 68.5.

### Question 9

- (i) Most candidates were able to obtain the correct value of  $N$ .
- (ii) Initial manipulation of the resulting equation was attempted correctly by most, but some candidates incorrectly attempted to use natural logarithms before any algebraic manipulation had taken place.
- (iii) Attempts at differentiation of the function were met with varying degrees of success. Some candidates were unable to correctly differentiate the exponential function with respect to  $x$  with errors involving  $\frac{1}{100}$  being the most common. Many candidates did not read the question fully and gave their final answer in terms of  $x$  rather than  $N$ .

Answer: (i) 250 (ii) 208 (iii) 4700

### Question 10

- (a) (i) Many candidates appeared to be unfamiliar with the notation used in this part of the question. Knowledge of this notation is part of the syllabus requirements. Many candidates either chose to leave their question paper blank or confused the notation with that of  $f^2$ .
- (ii) Most candidates were familiar with the notation for this part of the question and were able to obtain a correct answer.

- (iii) Many candidates recognised the notation and were able to attempt a correct method, often reproducing their work in part (i). Most equated their result to  $-1$  but there were many errors in the algebraic manipulation required. Some candidates misinterpreted the notation, choosing to work incorrectly with  $(f(x))^2$ , and a few attempted to work incorrectly with  $f^2(-1)$ . It is important that candidates have an awareness of the different types of notation that can be used when dealing with functions.
- (b)(i) Many candidates tried to express the given functions as a simple function rather than a composite function, clearly not understanding what was required of them.
- (ii) Candidates who had obtained a correct result for part (i) were usually able to obtain a correct result for part (ii), clearly being able to interpret what was required correctly. Only a few wrote their composite functions in the incorrect order.

Answer: (a)(i)  $2(2+x)^{-3}$  (ii)  $\frac{1}{x}-2$  (iii)  $-\frac{7}{3}$  (b)(i) gh (ii) kg

### Question 11

- (i) Most candidates were able to find the gradient of the line  $AB$  and the gradient of a line perpendicular to  $AB$ . Finding the coordinates of the point  $P$  caused difficulties with some candidates using the midpoint of the line in spite of being told where  $P$  was on the line  $AB$ . A significant number of candidates filled the allocated space with quadratic equations dealing with Pythagorean calculations for a third of the distance  $AB$ . Most were unable to produce anything meaningful from these equations. Very few candidates chose to make a sketch of the situation and consider the position of  $P$  in a logical manner using displacements parallel to the coordinate axes.
- (ii) Candidates who realised that  $y = 13$  were usually able to get a method mark, but many failed to appreciate the meaning of the phrase 'parallel to the  $x$ -axis' and used either  $x = 11$ , or  $y = 0$  or some intersection of two lines in previous working.
- (iii) The matrix method of finding the area of a triangle was used by the majority of candidates and was usually successful in gaining the method mark for this part of the question. Candidates who did not use this approach and chose to use Pythagoras' theorem did not always appreciate where the right angle was and, as a result, used incorrect lengths. Again, a simple sketch would have helped in many cases as the area could also be found simply using the horizontal and vertical lengths involved, rather than any more complex methods.

Answer: (i)  $2x + 3y = 9$  (ii)  $(-15, 13)$  (iii) 156

### Question 12 Either

Very few candidates chose to do this option. Of those that did, the majority usually scored highly.

- (i) Most candidates were able to obtain the required velocity vector and obtain the given result correctly.
- (ii) Having obtained a correct result from part (i), most, if not all candidates were able to obtain the position vector of the ship after  $t$  hours.
- (iii) The displacement vector of the ship at 16 00 hours was obtained by most candidates and often a correct velocity vector for the speedboat was obtained. The question specified the speed of the speedboat and often candidates left their final answer as a velocity.

- (iv) Many candidates still find relative velocity difficult. All that was required was an appropriate subtraction of the two velocity vectors. Many candidates chose to work with speeds rather than velocities thus highlighting the fact that many candidates do not really understand the difference between a speed and a velocity.
- (v) A correct approach gained the great majority of candidates at least the method mark.

Answer: (ii)  $54\mathbf{i} + 16\mathbf{j} + (12\mathbf{i} + 16\mathbf{j})t$  (iii) 64.8 (iv)  $39\mathbf{i} + 24\mathbf{j}$  (v)  $51.9^\circ$

### Question 12 Or

This question was done by most candidates with varying degrees of success. Few were able to gain full marks for this question.

- (i) Most candidates were able to obtain  $\overrightarrow{OQ}$  and  $\overrightarrow{PQ}$  with the occasional sign error or slip in simplification.
- (ii) A valid attempt at finding  $\overrightarrow{QR}$  in terms of  $\lambda$ ,  $\mathbf{a}$  and  $\mathbf{b}$  was made by most candidates with the occasional sign error or slip in simplification often from an incorrect result in part (i). This did not prevent candidates obtaining method marks.
- (iii) Very few candidates obtained a correct answer for this part of the question. The most common response was  $\overrightarrow{QR} = \mu \left( \lambda \mathbf{a} - \frac{5}{4} \mathbf{b} \right)$  which, although correct, is not in the form requested in the question, which required  $\overrightarrow{QR}$  to be in terms of  $\mu$ ,  $\mathbf{a}$  and  $\mathbf{b}$ .
- (iv) Many candidates realised that they needed to equate like vectors and went on to gain full marks by using their responses for parts (ii) and (iii). Candidates were not penalised if they used  $\overrightarrow{QR} = \mu \left( \lambda \mathbf{a} - \frac{5}{4} \mathbf{b} \right)$  from part (iii). Again, there were sign errors and errors in simplification.

Answer: (i)  $\overrightarrow{OQ} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ ,  $\overrightarrow{PQ} = \frac{2}{3}\mathbf{a} - \frac{11}{12}\mathbf{b}$  (ii)  $\overrightarrow{QR} = \left( \lambda - \frac{2}{3} \right) \mathbf{a} - \frac{1}{3}\mathbf{b}$   
(iii)  $\overrightarrow{QR} = \frac{\mu}{1-\mu} \left( \frac{2}{3}\mathbf{a} - \frac{11}{12}\mathbf{b} \right)$  (iv)  $\mu = \frac{4}{15}$ ,  $\lambda = \frac{10}{11}$

# ADDITIONAL MATHEMATICS

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Paper 4037/21

Paper 21

## Key message

In order to succeed in this paper, candidates need to have a full understanding of all the topics on the syllabus. They need to read the questions carefully in order to ensure that they are answering the question asked, using the required method and giving their answer in the form required, where these are specified.

## General comments

Presentation of answers was generally very clear to follow, the answer-book format helping the majority of candidates to organise their work. However, on questions which required solutions made up of several steps, it was difficult for Examiners to follow the order of some candidates' work. Where a question would benefit from a diagram – which is usually the case with vectors – then candidates should be encouraged to concentrate on producing as clear a diagram as possible. Candidates should take care in the accuracy of their answers and Centres would be advised to draw attention to the rubric which clearly states the requirement for this paper. While interim answers within working do not need to be stated to the same accuracy, the value being worked with should be as accurate as possible to avoid losing marks at the end of the question. Candidates should always try to take note of the form of the answer required and where a specific method is indicated be aware that little or no credit will be given for alternatives. Candidates should also be aware of the need to use the appropriate form of angle measure within a question as this can lead to all further work being invalidated.

Candidates scored well on **Question 1**, **Question 5**, and **Question 11**. The lack of any form of diagram on **Question 8(iii)** often led to work that was of little relevance and thus gained little credit. There were often incomplete solutions to **Question 4(iii)**, **Question 7** and **Question 9**. Those candidates who did not read **Question 6** carefully did not gain full marks despite accurate diagrams.

## Comments on specific questions

### Question 1

- (i) Almost all candidates gave a correct answer. The most common error was in calculating the determinant, and there were also errors with candidates getting the matrix elements the wrong way around.
- (ii) Despite the clear instruction, many candidates did not use their answer from part (i) to solve part (ii). When using the appropriate method in part (ii), the matrices were generally written in the correct order and evaluated correctly.

Answers: (i)  $\frac{1}{26} \begin{pmatrix} 5 & 3 \\ -2 & 4 \end{pmatrix}$  (ii)  $x = 0.5, y = 4$

### Question 2

This was generally well done, with the rationalisation method familiar to most candidates. Some candidates appeared to have used calculators, despite the instruction not to do so, as the correct answer sometimes appeared without a statement of the rationalisation step or clear evidence of its application. Candidates should be made aware that this is malpractice and could result in the loss of marks. A few divided the volume by  $(2+\sqrt{3})$  rather than its square. On the whole, manipulation of surds was well handled.

Answer:  $4 - \sqrt{3}$

### Question 3

In general, part (b) was found more accessible than part (a). A number of candidates made no attempt at any part of this question.

- (a) The approach to this part varied considerably with some candidates simply writing down values and others giving detailed working. A significant number of candidates gained 2 marks, with the most common error being made on calculating  $b$  (many gave an angle, or 4, as an answer). There were many candidates who were unable to score on this part at all.
- (b)(i) This was well answered by most, with the most common error being “3”. A small number of candidates reversed the answers to parts (i) and (ii).
- (ii) The correct answer was given by the majority of candidates. As in part (i) the more popular approach was to simply state the value required.

Answers: (a)  $a = 3$ ,  $b = 8$ ,  $c = 7$       (b)(i)  $\frac{2\pi}{3}$  or 120      (ii) 5

### Question 4

- (i) Those candidates who realised the need to use the product rule usually gained full credit. Partial credit for  $2x \ln x$  was also often awarded.
- (ii) Although the question stated ‘hence or otherwise’ the most successful method, unfortunately undertaken by relatively few, was to use the answer for part (i) and use the expected knowledge that differentiation and integration are inverse operations. This type of question has been set previously and needs to be drawn to the attention of candidates. A small number of candidates attempted to integrate by parts with limited success.

Answers: (i)  $2x \ln x + x^2 \times \frac{1}{x}$       (ii)  $\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 (+c)$

### Question 5

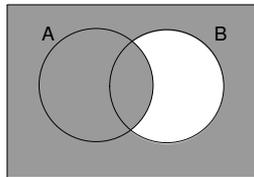
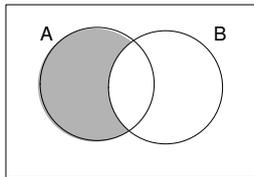
- (a) Many candidates gave completely correct solutions to this part, using a variety of bases for the taking logarithms step. The most common error was due to premature rounding, leading to 3.15 as the answer. Candidates were asked to give the answer to 2 decimal places and otherwise correct solutions could not gain full marks by omitting to do so. A few candidates made no progress in knowing how to deal with the indices.
- (b) The starting point for solving this type of question seemed to be well understood by the majority, with most converting all terms into powers of 6, occasionally with a loss in accuracy in multiplying out simple brackets. The next step was less well attempted although many did so successfully. This was usually as a result of dividing/multiplying the indices rather than subtracting/adding.

Answers: (a) 3.14      (b)  $y = -4.5$

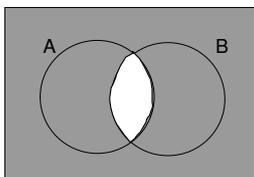
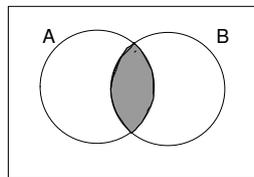
**Question 6**

Of the three parts, a fully correct answer was most likely to be seen in part (iii). It was not often that a correct second diagram was given in parts (i) or (ii). The second diagram commonly presented in part (ii) represented  $(X \cup Y)'$ . However, a number of candidates did not read the question carefully. Candidates were instructed to state their conclusions clearly but a significant number gave two correct diagrams with no indication of whether or not they understood the statement to be true or false. In cases like these candidates need to be aware that they must make a decision in order to gain full credit and not rely on the Examiner to assume their intentions.

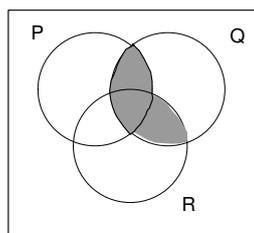
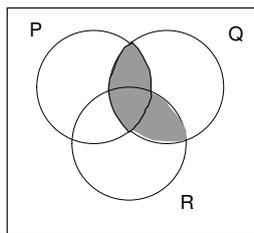
*Answer:*



And False



And False



And True

**Question 7**

A significant number of candidates did not realise that it was the second derivative that was required, presumably due to a lack of familiarity with the notation. These candidates frequently attempted to work with inverse functions instead. It was also common to see  $ff(x)$  or even  $fff(x)$  attempted. Of those that grasped what the question was asking, the majority answered the question well with an occasional error involving signs when differentiating negative indices. Candidates who realised the need to differentiate should also be careful not to apply the quotient rule too readily as this proved less successful, often as a result of differentiating  $648$  to  $1$  or even  $648$ .

*Answer:*  $x = 9$

### Question 8

The standard of knowledge of vectors amongst candidates appears very variable indeed. Even stronger candidates frequently obtained only partial credit, usually through not finding the magnitude of  $\overrightarrow{OC}$  in part (ii), and being unable to make meaningful progress in part (iii). Many candidates omitted this question.

- (i) This was the best attempted part of the question with many correct solutions. Some candidates gave an expression in terms of  $\overrightarrow{OB}$  and  $\overrightarrow{OA}$  or  $\overrightarrow{AO}$  without evaluating it, although this was then usually given in part (ii). A common error was to add rather than subtract the appropriate vectors.
- (ii) Where an answer was offered the candidate usually found a third of their answer from part (i) and added it to  $\overrightarrow{OA}$ . The length of this vector was frequently not found, however, either as an oversight or through a lack of understanding regarding the difference between a vector and its length.
- (iii) This part of the question proved very difficult with only a few completely correct answers. The simplest and most elegant solution was, possibly with the aid of a diagram, to realise by using similar triangles that  $D$  was two-thirds of the way along  $\overrightarrow{OA}$ .

Answers: (i)  $9\mathbf{i} + 45\mathbf{j}$  (ii) 13 (iii)  $\frac{4}{3}\mathbf{i} - 2\mathbf{j}$

### Question 9

This question differentiated well between candidates. Some candidates made little progress at all, others realised the need to differentiate to find acceleration but then substituted  $t=0$ , assuming this was instantaneous rest. Another group of candidates solved for  $v=0$  and substituted back into a variation of the expression for  $v$ . Of those candidates who understood what steps were required a large number, as in **Question 7**, were too quick to apply the quotient rule for differentiation with again less success than those who used negative indices. This was particularly the case with those who applied the quotient rule to a single fraction. However, a good number of fully correct solutions were seen.

Answer:  $\frac{11}{6}$

### Question 10

Good skills in coordinate geometry were often in evidence with a fair proportion of fully correct answers. Many candidates knew how to find the equations of  $AD$  and  $CD$ , and to solve them to find  $D$ . Errors which occurred were often in the algebraic manipulation, usually involving the incorrect use of signs. There were also some candidates who progressed little further than finding the gradient of  $BC$ , or who attempted to calculate or state the lengths of sides without finding  $D$ . A variety of methods was seen for finding the area, but the most common was to use the standard formula for the area of a trapezium. A common mistake was to use  $AB$  as the height rather than  $CD$ .

Answer: 55

### Question 11

- (i) This was generally well attempted with many correct solutions. As always with questions where the answer is stated, it is important not to use the answer in a circular fashion. Candidates are also advised to check their working and not assume it will give the correct value, as some who used  $\tan$  rather than  $\sin$  should have noticed.
- (ii) It was good to see that very few candidates reversed the formulae for perimeter and area. Candidates should be encouraged to work with radians and the corresponding formulae as conversion to degrees adds additional opportunity for slips and early approximations in calculation. Similarly candidates should avoid rounding 2.14... to 2.1 as using this value often lead to an inaccurate final answer. The overall plan for the perimeter involved more planning than the area and as a result fewer candidates gained full credit, either omitting a length or including an extra one.
- (iii) The plan was executed more consistently than in part (ii), the most common loss of marks being due to prematurely rounding values.

Answers: (ii) 54.3 (iii) 187

### Question 12 Either

- (i) This was the most successful part of the question, with many correct answers and a large number who achieved  $a$  and  $b$  or  $a$  and  $c$ . The most successful method was to expand and equate rather than by the more traditional completing the square route. Relatively few however, appreciated the significance of finding this form first as an aid to the remaining parts.
- (ii) This part was also quite well done but usually by differentiating the original expression rather than writing down the answer based on part (i). This proved a lot of work for 1 mark.
- (iii) This part met with very little success as most candidates attempted to manipulate the original expression, sometimes to find an inverse but often with no apparent target.
- (iv) The most common diagram was of a 'full' quadratic curve, sometimes accompanied by a 'full' inverse. Very few realised the restricted domain found in part (iii) needed to be applied.
- (v) Those working from the form of the equation obtained in part (i) were usually successful. However, far too many went back to the original form, found they could not rearrange it for  $x$  in terms of  $y$ , and resorted to a mixture of unconvincing algebra and the selective change of a variable in their expression to present an answer.

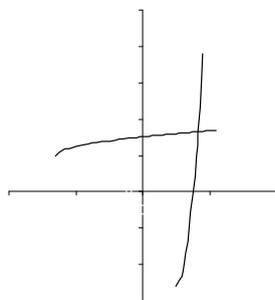
Answers: (i)  $y = 2(x - 5)^2 - 13$

(ii) (5, -13)

(iii) 5

(iv)

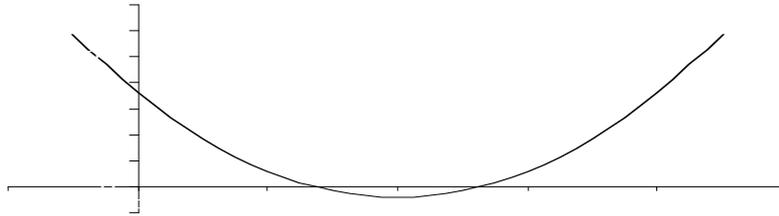
(v)  $5 + \sqrt{\frac{(x+13)}{2}}$



**Question 12 Or**

- (i) While expansion and equating proved the more successful method, completing the square was also prevalent and overall this part was completed successfully by most candidates.
- (ii) As in **Either**, candidates did not always appreciate the relevance of what they had already found. Those who did were usually successful save for some slackness in not applying the partial inequality.
- (iii) This graph proved more successful than that in **Either** as it did not have a restricted domain. In such cases candidates are encouraged to draw a more complete graph with some indication of how it appears in all quadrants through which it passes.
- (iv) This was generally a source of some marks for those that attempted it, particularly if they had completed part (i) correctly and then applied it. Omitting the negative root of the ln equation was a common error although it was good to see that most did attempt to find  $x$  and did not stop when they found  $\ln x$ .

Answers: (i)  $p = -40, q = -8$       (ii)  $g(x) \geq -8$   
(iii)



(iv)  $x = 7.39$  or  $e^2$ ,  $x = 403$  or  $e^6$

# ADDITIONAL MATHEMATICS

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Paper 4037/22

Paper 22

## Key messages

Candidates should be reminded of the need to show all their working. After any error, further marks can only be awarded if there is clear evidence that the correct method is being used.

## General comments

The spread of marks suggested that the paper provided good differentiation between candidates. All candidates appeared to have sufficient time to attempt all of the questions.

The standard of algebraic manipulation was good and **Questions 2** (small changes), **3** (conditions for a line to be a tangent to a curve), **5** (factor theorem, solution of a quadratic and manipulation of surds), **6** (binomial expansion) and **9** (solution of trigonometric equations) were very well answered.

**Questions 1, 8** and both of the options in **Question 11** proved to be very difficult for many of the candidates.

Centres are reminded that, from June 2013, there will be no choice of questions in the papers.

## Comments on specific questions

### Question 1

The notation required for this question appeared unfamiliar to many candidates. The symbols  $\subseteq$  or  $\subset$  were used more frequently than  $\in$  in part (a) and  $\varnothing$  was used instead of  $\notin$  in part (b). There was also frequent use of  $=$  and  $\neq$  sometimes combined with  $0$  or  $\phi$ . Solutions were regularly written between inappropriate brackets and an unnecessary "x" also appeared at times.

Centres would be well-advised to ensure that candidates are given more experience of this section of the syllabus.

Answers: (i)  $7 \in P$     (ii)  $8 \notin S$     (iii)  $n(N \cap S) = 6$

### Question 2

- (i) This question was very well answered with most candidates being successful. Some, who nevertheless were successful, made the question more difficult for themselves by writing  $\sqrt{(4x+1)^3}$  as  $((4x+1)^3)^{\frac{1}{2}}$  and treating it as a function of a function of a function instead of just working with  $(4x+1)^{\frac{3}{2}}$ . Others multiplied out the bracket.

- (ii) This part caused more problems for weaker candidates, some of whom did not replace  $x$  with  $6$ .

Answers: (i)  $6\sqrt{4x+1}$     (ii)  $30p$

### Question 3

The majority of candidates scored full marks on this question. Candidates who chose to eliminate  $y$  generally completed the solutions successfully, although there were a few who reversed the signs in the factorisation. Those candidates who chose to use calculus to find  $x$  and then find  $m$  were more likely to make errors.

A small number of candidates chose to equate the original equation for  $y$  to zero.

Answer:  $m = -3$  or  $9$

### Question 4

- (i) A pleasing number of candidates scored well on this question as they were able to decide how the information should be displayed in matrix form.
- (ii) Many candidates lost a mark by not identifying which contestant achieved which total score. Not all candidates with the correct matrices obtained the correct answers; often it was numerical carelessness which cost them marks.

Answers: (i)  $\begin{pmatrix} 4 & 1 & 7 \\ 2 & 5 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 & 5 & 2 \\ 4 & 3 & 6 \end{pmatrix} \begin{pmatrix} 8 \\ 4 \\ 2 \end{pmatrix}$  (ii) Claire 70 and Denise 82

### Question 5

- (i) The vast majority of candidates used the factor theorem to answer this question and provided clear setting out of solutions. This part was well answered, even by the weakest candidates.
- (ii) The majority obtained the correct quadratic, some used division, others comparison of coefficients, the latter producing more errors. Many went on to use the formula correctly. A few gave the final answer as a decimal, but a pleasing number were able to manipulate the surd and arrive at the required form.

Answers: (i)  $k = 4$  (ii)  $-3 \pm \sqrt{5}$

### Question 6

- (a) (i) This was another question that was particularly well answered, with the vast majority of candidates being successful. The most common cause of error was the lack of use of brackets, meaning that  $-2x^3$  was used instead of  $-8x^3$ .
- (ii) Again, there were many completely correct solutions, although not all candidates calculated and collected the two required terms.

Candidates generally answered part (a) successfully, although many did extra work calculating terms other than the ones required.

- (b) This was not as well answered as part (a), but most candidates who identified the correct term usually went on to calculate it correctly. A number were able to work out that  $r = 2$  gave the correct term but were then unable to go on from there. Inevitably there were candidates who did not understand "term independent of  $x$ ".

Answers: (i)  $-280$  (ii)  $-504$  (iii)  $135$

### Question 7

Some candidates did not recognise that the expression could be dealt with using logarithms and therefore did not score very highly on this question. Of those who did, most went on to draw a satisfactory graph and calculate the required values.

- (i) Candidates who realised the need to set up a log/log graph either in base 10 (by far the more popular) or base  $e$  usually went on to score very well throughout the question. Far too many candidates, however, were uncertain of how to proceed. A great many simply plotted the points as they appeared in the question. They then either joined them with a curve, a series of straight line sections or they attempted to produce a line of best fit.
- (ii) Without a correct attempt at a graph in part (i) candidates could make little progress in part (ii). Almost all candidates realised that  $a$  and  $b$  followed from the gradient of the line and the intercept on the  $y$ -axis. A small number of candidates managed to get them the wrong way round and a number of others thought that  $a$  was the intercept rather than  $\log a$ .
- (iii) Candidates with the correct graph usually went on to find the correct answer in this part, either by calculation from the original equation or by appropriate readings from their graph. Others were able to gain a mark by correctly substituting their values from part (ii) into the equation.

Answers: (ii)  $a = 4 \pm 0.3$ ,  $b = 0.5 \pm 0.03$  (iii) 40

### Question 8

Many candidates had been well prepared for answering a minimising problem and showed clarity of thought by their clear and accurate presentation.

However, other candidates were less well-prepared and would benefit from more work on this type of question. There were a number of common errors. Some candidates lost the last mark due to misinterpreting “dimensions”; giving the minimum area rather than working out the other dimension. There were solutions where a non-algebraic approach was taken but having ‘guessed’ the dimensions there was no justification as to why they would give the minimum surface area. Some candidates did not differentiate but equated an expression for the surface area to zero. Others differentiated an expression for the surface area in two variables but treated one of the variables as a constant.

A number of candidates, however, seemed uncertain as to how to attempt this problem. Of the weaker solutions, many only gained 1 mark, for writing a volume equation correctly. It was apparent that some did not know how to find the surface area of a cuboid.

Answer:  $8 \times 8 \times 4$

### Question 9

The first two parts of this question were particularly well answered. As usual, candidates had less success when dealing with radians in the final part.

- (a)(i) This was answered correctly by most candidates although some had a tangent value which was the reciprocal of the correct value. The majority got both solutions but occasionally the second solution had not been identified.

The minority of candidates who squared both sides had extra solutions so lost 1 mark. A few candidates replaced  $\sin x$  with  $1 - \cos x$ .

- (ii) Many candidates got this totally correct with well-presented solutions, although some gave 78.5 as an extra solution.

- (b) Most candidates realised that this was in radians but a few worked in degrees.

Many might have produced a clearer solution had they written both possible angles at the " $3 - z =$ " stage. Not many showed that they had used  $\pi - 0.927$  even when they obviously had.

Some lost marks due to working to an insufficient degree of accuracy.

Answers: (a)(i) 59(.0), 239(.0) (ii) 101.5, 258.5 (b) 0.927, 0.786

### Question 10

Part (a) was answered well, but part (b) was found more difficult. A number of candidates were uncertain whether to use permutations or combinations. Those who tried to use reasoning frequently performed better.

- (a) (i) This was very well answered by the vast majority of candidates although there was a minority of candidates who chose to use permutations rather than combinations.
- (ii) Again, generally very well done. A number of candidates did not appreciate that there were only 5 women available and included 6 and 1 and even occasionally 7 and 0 as well as the two correct combinations. As with part (i) some chose to use permutations rather than combinations.
- (b) (i) This part and the next almost always scored 2 or 0. Candidates who thought the problem through logically and wrote down the correct multiplication got the correct answer, and this was a great many of the candidates. Those who tried to introduce permutations and combinations rarely made progress.
- (ii) Not as well done as the previous part but again those who approached the problem logically were successful.

Answers: (a)(i) 792 (ii) 196 (b)(i) 240 (ii) 48

### Question 11

11 **EITHER** was chosen by about 70% of candidates and generally those candidates who chose **EITHER** scored more marks than those choosing **OR**. Candidates who chose not to use exact values but to use decimal approximations frequently lost some marks because of premature approximations.

#### **EITHER**

- (i) Almost all candidates found the derivative of  $y$  correctly although a few did not remember to pre-multiply by  $\frac{1}{2}$ . Almost all tried to find the gradient, set up the correct equation and solve. Far too many lost accuracy in calculating the  $x$ -coordinate of  $Q$ . Those who worked, and gave their answer, in terms of  $\pi$  were much more successful.
- (ii) All but the weakest of candidates realised that the basic strategy required them to find the area of the appropriate triangle and subtract the appropriate area under the curve. Most candidates successfully used a correct method to calculate the area of the triangle. Candidates who realised that they needed to find where the curve cut the axis ( $2\pi$ ) and use that as a limit along with  $3\pi/2$  usually went on to calculate an acceptable answer for the area under the curve.

A significant number of candidates chose to use the idea of  $\int (f(x) - g(x)) dx$ ; unfortunately many of these used a common upper limit, making the method flawed.

Answers: (i)  $y = 0$ ,  $x = \frac{3\pi}{2} + 2$  or 6.71 (ii)  $\frac{3\sqrt{2}}{2} - 2$  or 0.121

**Question 11 OR**

- (i) Most candidates could differentiate correctly but many were unable to see the connection between the “show that” question and the differentiation they had just done. The best answers to the “show that” part gave sufficient steps in their working out to clearly demonstrate their understanding.
- (ii) The better candidates were able to manipulate the algebra and work at all times in terms of  $e$ . This led to neat, accurate and often quite succinct solutions. Many chose to integrate to find the area under the line rather than working out the trapezium area. Weaker candidates had no clear plan and appeared to bounce around between different approaches leading to confusion and incorrect answers. Some candidates after writing down the integral of the line then forgot to integrate it and substituted the limits into the equation of the line.

Some candidates, when integrating for the curve, showed no evidence of using the limits, just giving an answer. This was a problem when their numerical answer was incorrect as there was no evidence for the method mark. The majority of candidates used the integral from part (i) but some candidates did not substitute the limit of zero correctly.

Candidates working in decimals rarely got full marks due to losing the last accuracy mark.

Answers: (i)  $(1 - x)e^{-x}$       (ii)  $\frac{9}{e^2} - 1$  or 0.218