ADDITIONAL MATHEMATICS

Paper 4037/11

Paper 11

Key Messages

Candidates should be reminded of the need to present their work clearly and concisely in the spaces provided on the question paper. Calculators should always be checked to make sure that they are in the correct mode required for a given question and that working is carried out to the required level of accuracy. Candidates should also ensure that they follow the instruction when a question uses the word 'hence'.

General Comments

The examination gave candidates plenty of opportunity to display their skills. There was no evidence that the examination was of an inappropriate length and few candidates omitted questions or parts of questions. Many candidates gained high marks, showing a good understanding of the syllabus and its applications. Marks were at times lost due to prematurely rounding or truncating results.

Comments on Specific Questions

Question 1

(i) Most candidates realised that this was one cycle of a cosine curve translated downwards by one unit. Although this was only a sketch, candidates were expected to show the start of a turning point at both $x = 0$ and at $x = 2\pi$. However, many candidates did not do this and sketched the curve as a parabola.

(ii) A number of candidates plotted the curve at intervals of $\frac{\pi}{2}$ and thought that the curve was a straight line through those points on the x-axis.

(iii) Candidates who had made reasonable sketches in parts **(i)** and **(ii)** often did not take account of the intersections at 0 and 2π and gave the incorrect answer of 1 solution.

Answer: **(iii)** 3

Question 2

Most candidates were able to find the gradient and the $\ln y$-intercept of the straight line, but many were unable to make any further progress. There were a number of possible alternative methods of solution, substituting values in either the logarithmic or the index form of the equation, but these were often confused with errors such as $Ab^2 = 4$ being typical.

Answer: $A = 7.39$, $b = 2.72$

Question 3

All three parts of this question were usually done well and the majority of candidates obtained full marks. Only a few attempted to use permutations instead of combinations.

Answer: **(i)** 3003 **(ii)** 840 **(iii)** 2919

Question 4

(i) The majority of candidates were able to find *x*, although, with those who did not appreciate that the logarithm of a negative number does not exist, the answer ±2 was quite common.

(ii) The laws of logarithms were generally well-known, although some candidates thought, erroneously, that the required quadratic was $y^2 - (5y - 12) = 2$. Some of those who did not notice the link with part **(i)** produced the equation $y^2 \div (5y - 12) = \frac{1}{2}$.

Answer: **(i)** 2 **(ii)** 4, 6

Question 5

(i) The integration was generally carried out well, although a number of candidates thought that $6x^{-1}$ simplifies to $\frac{1}{6x}$ causing them to lose marks in part **(ii)**.

(ii) Limits were usually dealt with correctly, but a number of candidates produced a cubic equation, rather than a quadratic, which they were often unable to deal with.

Answer: **(i)** $x + \frac{6}{x}(+c)$ **(ii)** $k = 2$

Question 6

(i) The process for finding the inverse of a 2 x 2 matrix was well-known.

(ii) Most candidates knew how to multiply matrices. For those who used the answer from part **(i)**, the most common error was to post-multiply the right hand side by $\mathbf{A}^{-1}$ rather than pre-multiply. Those who answered the question using the alternative method were generally successful with only arithmetic errors leading to loss of marks.

Answer: **(i)** $\frac{1}{13}\begin{pmatrix} 5 & 1 \\ -3 & 2 \end{pmatrix}$ **(ii)** $a = 4$, $b = 2$, $c = 1$, $d = 1$

Question 7

(i) Most candidates got the correct ratio for $\tan B$ and proceeded to rationalise correctly. Those showing no working for the rationalisation process could not be awarded all of the marks. Where candidates are not allowed to use a calculator in a question, sufficient working needs to be shown to convince the Examiner that a calculator has not been used.

(ii) The majority of candidates adopted the easiest route to the answer, by using $\sec^2 B = 1 + \tan^2 B$. A number of candidates used $\sec^2 B = \frac{1}{\cos^2 B}$ which entailed much more work.

Answer: **(i)** $7 + 3\sqrt{5}$ **(ii)** $95 + 42\sqrt{5}$

Question 8

(i) Most candidates showed sufficient working for full marks.

(ii) A number of candidates appeared not to know the radian formulae, and answered this question using degrees, which was permitted.

(iii) As with the rest of this question, this was a good source of marks for many candidates.

Answer: **(ii)** 54.55 **(iii)** 115.25

Question 9

(i) Many candidates interpreted the ratio on AB correctly and were able to use vector addition to find $\overrightarrow{AP}$. A few used the given answer for $\overrightarrow{OP}$ to find $\overrightarrow{AP}$, which was not allowed.

(ii) Candidates found it more difficult to interpret the ratio given on OC correctly and, consequently, $\overrightarrow{OQ} = \frac{2}{3}c$ was common and incorrect.

(iii) This was generally answered well, with only a few candidates thinking that $\overrightarrow{BC}$ was either **b** – **c,** or **b** + **c**.

Answer: **(i)** $\frac{3}{4}(\mathbf{b} - \mathbf{a})$ **(ii)** $\frac{2}{5}\mathbf{c} - \frac{1}{4}\mathbf{a} - \frac{3}{4}\mathbf{b}$ **(iii)** $\frac{9\mathbf{b} - 5\mathbf{a}}{16}$.

Question 10

(i) The majority of candidates were aware of the need to differentiate, but a significant number found the equation of the normal rather than the tangent.

(ii) Virtually all candidates were aware of the need to solve the equation of the line with that of the curve and most were well-acquainted with methods for solving the resulting cubic equation.

(iii) A number of candidates did not understand the term "perpendicular bisector" and instead found the equation of the perpendicular through A.

Answer: **(i)** $y = 1 - 3x$ **(ii)** (0, 1) **(iii)** $3y = x - 7$

Question 11

(a) Most candidates found arcsin(–0.5) leading to one correct answer. A second correct angle within the required range was often omitted.

(b) Those candidates who chose to obtain a quadratic in $\tan y$ were generally successful in obtaining full marks for this part, the only common error being premature approximation to lead to an incorrect second angle. Candidates who substituted for $\tan y$ in terms of $\sin y$ and $\cos y$ were unable to make any progress.

Answer: **(a)** $\frac{5\pi}{6}, \frac{3\pi}{2}$ (2.62, 4.71) **(b)** 67.5°, 157.5°

ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key Messages

Candidates should be reminded of the need to present their work clearly and concisely in the spaces provided on the question paper. Calculators should always be checked to make sure that they are in the correct mode required for a given question and that working is carried out to the required level of accuracy. Candidates should also ensure that they follow the instruction when a question uses the word 'hence'.

General Comments

The examination gave candidates plenty of opportunity to display their skills. There was no evidence that the examination was of an inappropriate length and few candidates omitted questions or parts of questions. Many candidates gained high marks, showing a good understanding of the syllabus and its applications.

Marks were at times lost due to prematurely rounding or truncating results. Clarity of numbers and lettering would also have helped as sometimes it was difficult to distinguish between x and n, as well as 6 and 8. There were also marks lost due to carelessness with signs on terms, especially when expanding brackets and manipulating terms in equations.

Comments on Specific Questions

Question 1

Many candidates are still not fully familiar and comfortable with set notation and found it difficult to apply their limited knowledge to a scenario involving a subset.

(i) This part was the most successfully done, but not all candidates realised that if B was a subset of A, then $A \cap B$ would be equal to B.

(ii) The answer 11 was often given for part **(ii)** as not all candidates were aware of the relationship between A' and A.

(iii) Use of the diagram provided would have been particularly helpful in this part, as the question had been structured to lead them to an answer here. However, not all candidates seemed to use their diagram and at this final stage tended to add and subtract various combinations of the figures given in the question.

Answer: **(i)** 5 **(ii)** 16 **(iii)** 11

Question 2

All the parts of this question could have been easily answered from a basic approach setting out the digits in tabular form by just listing how many terms were available for each position. Had this been adopted, many would have been more successful than trying to attempt a method involving permutations and combinations as often the latter was used in the wrong situation. Those who used this logical method usually got all the answers correct.

(i) This was straightforward and mostly correct, but there were some candidates who used ${}^6C_4$ rather than the required ${}^6P_4$.

(ii) Whilst most candidates could spot the need to have the digit 5 in the final place, some multiplied this value into their result, producing 300 instead of 60. Some thought that as the number had to be divisible by 5, they just divided their answer to part **(i)** by 5. Once the digit 5 had been assigned to the final place, this should leave only 5 digits for the other 3 places but many insisted on selecting from six choices.

(iii) Not all candidates appreciated that there were 3 potential first digits and only used 7 in that place, resulting in many offering 1 x 4 x 3 x 1 or even worse 1 x 4 x 3 x 5. Where the first and last positions were assigned correctly, many still used six choices for the final 2 places and some that did realise there were only four remaining tried to use ${}^4C_2$.

Answer: **(i)** 360 **(ii)** 60 **(iii)** 36

Question 3

In questions of this type, clarity of presentation is paramount. When elements are eliminated it is helpful to ensure that lines of 'cancellation' do not obscure the terms so that they can be checked. The key word in this question is 'Show', so all relevant steps need to be included clearly. In general though, many candidates produced completely correct solutions. However, some had trouble expanding $(1-\cos\theta-\sin\theta)^2$. Trying to do this in two parts often led to errors, usually involving signs, so going for a full expansion to 9 terms was invariably more successful. A method mark was awarded for a correct use of $\cos^2\theta+\sin^2\theta=1$, but quite often candidates used this without making it clear they had done so.

Question 4

Most candidates started their solution to this question by considering $b^2 - 4ac$, although many were not clear that for the curve to lie above the x-axis they required $b^2 - 4ac < 0$. The most common mistakes at this stage were to take b as $(k + 2)$ and c as -6. Most candidates were able to obtain the two critical values of 4 and 12, but many were then unable to obtain the required inequality. A simple sketch of the quadratic function $k^2 - 16k + 48$ showing the two points $k = 4$ and $k = 12$, with the curve lying below the horizontal axis between the roots should have enabled the correct set of values of k to be chosen. Only a few candidates tried alternative methods of solution, using either completing the square or differentiation. These were usually much less successful with few being able to complete their solutions successfully or even arrive at the two critical values 4 and 12.

Answer: $4 < k < 12$

Question 5

Many candidates were able to produce completely correct solutions to this question, with most candidates making a good attempt and showing an understanding of what needed to be done.

Most candidates were able to substitute to obtain an equation in a single variable but some made errors in factorising the resulting quadratic, usually due to sign errors or the wrong combination of constant terms. The latter part of the question was also well answered, for those candidates who had found two sets of coordinates.

Answer: 1.25

Question 6

The mathematical processes needed for this question seemed to be well understood by the majority of candidates. Most candidates realised that they needed to differentiate and produced the correct derivative. Many candidates had problems calculating the values of y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ correctly for $x = \frac{3\pi}{4}$, but most attempted to use the negative reciprocal for the normal and went on to form the equation of the normal. Fraction calculations were handled confidently to give the exact answer or the answer correctly rounded to 3 significant figures.

Answer: –4.61

Question 7

(i) This part was generally well done with most candidates obtaining full marks. A few candidates assumed b to be 40 and only substituted it into one of the possible equations. Whilst they could obtain a this way, this value was not often checked in the other equation and b was rarely confirmed in this second equation as well.

(ii) Again, most candidates were able to obtain the correct solution to this part.

(iii) Candidates were expected to offer $x = -2$ as one solution and show that the remaining quadratic could not be solved. It was not essential that they said so, but merely showed an attempt at the formula that had a negative discriminant. However, many chose to ignore the negative value they had produced and insisted on producing a 'result'. Candidates should be reminded not to try to manipulate their work incorrectly in order to obtain results that they think they should have. Marks are invariably lost when this approach is adopted.

Answer: **(i)** –14 **(ii)** $(x+2)(6x^2 - 17x + 20)$ **(iii)** $x = -2$

Question 8

Overall this was very well attempted by most candidates with the majority of the errors being arithmetical only.

(a) (i) This part was usually done well, although there were a few who merely squared each element of the given matrix.

(ii) Some candidates went astray in this part by offering an incorrect version of the identity matrix, or by only multiplying the top left hand term by 4.

(b) (i) This part was virtually always correct with the exception of the occasional incorrect determinant.

(ii) A large number of candidates ignored the word 'hence' and so did not solve the equations by the matrix method as instructed. Consequently, they were restricted as to the marks they could earn. A few tried to post-multiply by their inverse which led to confused and incorrect solutions.

Answer: **(a)(i)** $\begin{pmatrix} 22 & -2 \\ -3 & 31 \end{pmatrix}$, **(ii)** $\begin{pmatrix} 16 & 6 \\ 9 & -11 \end{pmatrix}$, **(b)(i)** $\frac{1}{27}\begin{pmatrix} 3 & -1 \\ 9 & 3 \end{pmatrix}$, **(ii)** $x = \frac{1}{2}, y = 2$

Question 9

(i) Most candidates managed to apply the given formula for a binomial expansion to the question but many were unable to resolve the combinations, factorials and powers of 1. Few candidates obtained a correct third term with a few more being able to obtain a correct second term. Clearly, candidates used to numeric applications rely heavily on calculators to obtain terms in a binomial expansion and so may rarely have seen an algebraic approach.

(ii) A pleasing number of candidates recovered to obtain the three method marks but still had problems with expressing the coefficients in terms of *n*. Some good algebraic solutions were seen but many candidates who obtained full marks had resorted to the fairly straightforward trial and improvement method or an inspection of Pascal's triangle.

Some otherwise good solutions were marred by a careless loss of the minus sign from the $-{}^{n}C_{1}x^{2}$ term or the loss of the 4 from the term involving ${}^{n}C_{2}$.

Answer: **(i)** $1+\frac{nx}{2}+\frac{n(n-1)x^{2}}{8}$, **(ii)** 10

Question 10

(a) (i) This part was done correctly by most candidates, although some had a solution double that which was correct. A few integrated $2x-5$ as part of their answer.

(ii) Most candidates were able to use the limits correctly, even if it was on an incorrect expression, although some lost marks due to arithmetical errors.

(b) (i) Most candidates were able to differentiate a product correctly but unfortunately a few candidates erroneously combined $3x^{2}\ln x$ into $\ln 3x^{3}$ before use in part **(ii)**.

(ii) This part of the question caused many difficulties. Quite often candidates did not appreciate that by using their solution to part **(i)** (use of the word ‘Hence’ again), equated to $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{3}\ln x\right)$ and rearranging, the solution would be within reach. Integrating appropriately and dividing through by 3 thereafter was all that was required.

Answer: **(a)(i)** $\frac{1}{3}(2x-5)^{\frac{3}{2}}$ **(ii)** 41.3 **(b)(i)** $x^{2}+3x^{2}\ln x$ **(ii)** $\frac{1}{3}x^{3}\ln x-\frac{1}{9}x^{3}\ (+c)$

Question 11

(a) Most candidates knew that $\sec 2x$ should be replaced by $\frac{1}{\cos 2x}$ and the resulting equation rearranged, although solving (cos2x + 1)(cos2x + 2) was not always done correctly. There were some candidates who seemed to think that $(\cos 2x)^2$ was the same as $\cos^2 4x^2$. Far too often it proved difficult to solve cos 2x = –1 with many candidates gaining spurious solutions. Of those that were able to solve cos 2x = –1 correctly, many only gave one solution.

(b) There were many instances of completely correct solutions, showing a good understanding of the importance of order when solving trigonometric equations. However, many candidates made the incorrect initial move of $\sin^2 y - \sin^2 \frac{\pi}{6} = \frac{1}{2}$. Several other poor appreciations of the order required to correctly find y were seen. Some reached $\sin\left(y - \frac{\pi}{6}\right) = 0.707$ or equivalent, but then did not seek the inverse sine and tried to offer $0.0707 + \frac{\pi}{6}$ as their solution.

Answer: **(a)** 90°, 270° **(b)** $\frac{5\pi}{12}, \frac{11\pi}{12}$

Question 12

(i) and (ii) It was surprising how many candidates obtained the same answer for parts **(i)** and **(ii)** and did not seem to consider they may have missed something. Perhaps the concept of 'when the velocity stops increasing' was not properly understood, as candidates seemed to assume this meant the same as 'instantaneous rest'. Both answers were often given as t = 12, though there were times when both answers were given as t = 6.

(iii) This part was completed much more successfully, perhaps as this required the integration of the given function. It was good to see some candidates use an arbitrary constant and explain why this was zero.

(iv) Many candidates had problems with this last part of the question with many of those who did parts **(i)**, **(ii)** and **(iii)** correctly not realising that variable motion was still being considered.

There were too many instances of the use of either 'distance = velocity x time' or the equations of linear motion. However, for those candidates that did use calculus, there was a clear understanding about the difference between speed and velocity, and most candidates took the extra step from the negative result for the velocity, to give the positive value for the speed.

Answer: **(i)** 6 **(ii)** 12 **(iii)** 864 **(iv)** 324

ADDITIONAL MATHEMATICS

Paper 4037/21
Paper 21

Key Messages

Where an answer is given and a proof is required, candidates need to be aware of the need to fully explain their reasoning and not jump to the answer. Where a question would benefit from a diagram, which is always the case with relative velocity, then candidates should be encouraged to concentrate on producing as clear a diagram as possible.

General Comments

Questions which required the knowledge of standard methods were done well. More care needs to be taken when a good diagram would aid solution. Fully explained solutions were more successful on questions where the answer was given and required proving. Some candidates produced high quality work displaying wide ranging mathematical skills, with well presented, clearly organised answers. Presentation of answers was generally very clear to follow, the answer-book format helping the majority to organise their work. The majority of candidates made an attempt at most questions. There appeared to be more questions which candidates found accessible, possibly as a result of advice given in previous reports. Overall there was a full range of abilities in evidence. The majority of candidates, as always, attempted most questions, but at times some produced mathematics that had little connection to the question being attempted. These candidates need to improve their reading of questions and keep their working relevant. Candidates should take care in the accuracy of their answers and Centres would be advised to draw attention to the rubric which clearly states the requirement for this paper. While interim answers within working do not need to be stated to the same accuracy, the value being worked with should be as accurate as possible to avoid losing accuracy marks at the end of the question. Candidates should always try to take note of the form of the answer required and where a specific method is indicated be aware that little or no credit will be given for alternatives. Candidates should also be aware of the need to use the appropriate form of angle measure within a question as this can lead to further work being invalidated.

There was no single question which the majority of candidates found entirely straightforward, although there were several where candidates seemed able to score most of the marks with some regularity; **Questions 2**, **7**, **8** and **12** fell into this category. While candidates generally seemed able to apply the quotient rule for differentiation well in **Question 8** there was a much poorer understanding of the chain rule on **Question 3.** The lack of an appropriate diagram on **Question 10** often led to incoherent work which gained little credit, despite similar questions having appeared on recent examinations. There were otherwise no questions where most candidates struggled, but marks were frequently lost on **Questions 1, 4(i)**, **9(ii)**, **9(iii)** and **11(v)**. On both **Questions 1** and **5(i)** full marks were infrequently gained particularly as candidates often tried to use the given answer to justify their solution leaving it incomplete.

Comments on Specific Questions

Question 1

Some very succinct proofs were produced, but this was an opening question on which many struggled. Too often at the outset, the assertions that $(1+\sin\theta)^2 = 1+\sin^2\theta$ and $(1-\sin\theta)^2 = 1-\sin^2\theta$ or $1+\sin^2\theta$ were made. While an apparently correct solution could follow from this, it could never be fully correct and highlights the problem of candidates working towards a given result without giving due care to their method. Many candidates were able to gain some credit for their knowledge of trigonometric identities, but these need to be stated at a valid stage of a solution and not merely listed alongside or stated underneath an incomplete solution. Combining two expansions and then converting to individual fractions was the most successful method, but many also converted into sec and tan forms before expansion. While starting from both sides separately proved successful on occasion, too often this was combined with terms moved from side to side which candidates need to be advised is bad practice by assuming the result is correct in order to prove it.

Question 2

This was very well done, and there were many fully correct answers.

(i) The most common error in this part was to multiply rather than divide, but this was rare.

(ii) A few candidates tried to calculate a distance rather than a time.

(iii) The most common method was to split the trapezium into two triangles and a rectangle. The trapezium formula when used was usually applied correctly. When splitting the shape, a few candidates assumed the triangles were of equal size and there are still some who omit the division by two when finding the area of a triangle.

Answers: **(i)** $3.2\ \text{ms}^{-2}$ **(ii)** 15 s **(iii)** 312 m

Question 3

A good proportion of the candidates clearly understood the three part structure of the solution to this problem although there were a significant number who substituted 0.2 or 8 into their derivative. Finding $\frac{dy}{dx}$ proved difficult for the vast majority of candidates, with the most common error being to give it as $k\cos^2 x$. Only a few worked incorrectly in degrees and of those who found a correct derivative and value for x, correct working should have given an exact answer. On this occasion poor notation was not penalized but candidates should avoid using delta notation when the chain rule is called for.

Answer: –0.8

Question 4

(i) A majority of candidates understood what the modulus graph should look like. Unfortunately, many did not follow the clear instruction in the question on marking the points where the graph crosses the axes, with the $\left(\frac{1}{2}, 0\right)$ frequently being unmarked. A number of other candidates could identify the points of intersection correctly but often with unusual and incorrectly shaped graphs. Candidates who draw the original line and use it to derive the modulus would be advised to make it clear what they intend to be the final answer as a mixture of full and dotty lines can prove confusing and unproductive.

(ii) Not all candidates appreciated that there was more than one solution to the equation. Of the majority who did, errors were often made in assuming that the second solution is the negative of the first, or that just one of the signs of the expression inside the modulus sign has to be changed in forming an equation to find the second solution. A few squared both sides and solved the resultant quadratic, which was generally done well by those attempting it this way.

Answers: **(i)** $\left(\frac{1}{2}, 0\right)$, (0, 2) **(ii)** $\frac{2}{3}$, $\frac{2}{5}$

Question 5

(i) There were a large number of good solutions to this part. However, it would have been pleasing to have seen more candidates showing clearly where *QR* or *PS* came from, especially as the answer was given. There were also many attempts which tried to make their working fit the result and those who, perhaps in confusion, differentiated. Candidates generally perform better on questions where the result is given if they use the result as a comparison at the end of a solution rather than trying to work towards it.

(ii) Most candidates were aware of the need to differentiate and to set this expression to zero, which was generally also completed with accuracy. Unfortunately a significant number lost a mark by either a lack of careful reading or by misunderstanding. The error was generally not to find the optimum value of A assuming that showing that it was a maximum was all that was required, although a few found A without stating its nature.

Answer: **(ii)** $A = 384$, maximum

Question 6

This question was generally answered well, with most candidates showing the knowledge of both applying the quotient rule and of finding the equation of a normal. The common error usually occurred as a sign slip when sorting out the quotient rule, often due to omitting brackets around the $(x^2 + 8)$. Most candidates applied the quotient rule rather than the product rule, and generally it was more successfully used than the product rule. It was rare to see the equation of the tangent given and most solutions were complete and clear.

Answer: $y - 12 = \frac{1}{2}(x - 4)$ or $y = \frac{1}{2}x + 10$

Question 7

(i) This part was exceedingly well done, even by apparently weaker candidates. The majority of candidates simplified all of the terms, with C notation rarely being left as a final answer, which is a clear improvement on previous similar questions.

(ii) Success was more likely to be achieved by those who first multiplied out the two additional brackets. Those who did not, but either multiplied separately by the two additional brackets, or tried by inspection of the given expression to list the terms directly, were less likely to succeed. Many candidates did, however, insist on giving more terms than required, which is acceptable when they identify the term(s) requested but is likely to lead to uncertainty when not selected or when a slip was made in expanding to a quadratic.

Answers: **(i)** $64 + 192x + 240x^2 + 160x^3$ **(ii)** 64

Question 8

This was another straightforward question for many candidates, and a good number of fully correct answers were seen. Few seem to have had difficulties in knowing what to do and, where marks were lost, it was usually due to errors in the algebra most frequently leading to an incorrect quadratic. Most candidates substituted for y to get a quadratic in x. This was inevitably more successful than rearranging and substituting for x. A small number of candidates appeared to spot the implicit nature of the given equation and assumed that this was a question requiring implicit differentiation. As always, candidates are advised to read questions thoroughly before attempting them as accurate work gains no reward if it is irrelevant.

Answer: 22.4 or $\sqrt{500}$ or $10\sqrt{5}$

Question 9

In spite of the clear information given in the question, a majority of candidates assumed that sets of integer values were required. Candidates are advised, as has been mentioned in previous reports, to look carefully at the full definition of the sets before starting to solve. The majority, if not all, of the marks could be lost by this error.

(i) This part was the best attempted with most candidates being able to identify at least the correct solutions to the quadratic equation. The correct inequality did not always follow and there seemed to be little by way of method in this step with the inequality(ies) usually just being stated.

(ii) Many candidates were able to identify the range for set T but very few could combine this correctly with S often not realizing that T had an upper limit of 12 and not 8.

(iii) In the main, this part was not well attempted with few being able to identify the relevance of the complement. Of those that did this was too often in purely integer form. Where candidates clearly identified $S \cap T = 4 < x < 8$, or an attempt at this, more progress was likely to be made. Some confusion with strict/non-strict inequalities was evident from those that were able to successfully answer the question.

Answers: **(i)** $-3 < x < 8$ **(ii)** $S \cup T = -3 < x < 12$ **(iii)** $-5 < x \le 4$, $8 \le x < 12$

Question 10

Unfortunately relative velocity remains a topic which the majority of candidates find extremely difficult. The key to solving a relative velocity question is to produce a correct vector diagram at the start, but few could do this. Many got no further than the diagram, and a significant number produced a great deal of working which warranted little, if any, credit due to their incorrect diagram. Candidates must realize that these questions do not generally lead to right-angles triangles and therefore basic trigonometry and Pythagoras are inappropriate. Similarly, adding the given velocities or trying to combine displacement and velocity on one diagram is also not likely to gain any credit. When drawing diagrams, candidates should avoid both vectors starting at a common point and consider the basics of vector addition.

Answers: **(i)** $(0)21.8$ or $(0)22$ **(ii)** 114 or 1 hour 54 mins

Question 11

On the evidence shown, candidates in general displayed a varied ability to cope with the many aspects of functions. Only the best candidates scored full marks across the question and there were cases of candidates omitting the question completely.

(i) Answers for this part were mixed. With only one mark available, the expectation was that candidates would identify the correct value from the bracket. While this was the case for many candidates there were also large numbers who gave 0, 5, –5, –1 or a range for *k*.

(ii) As with **(i)** this met with a mixed response. Among the errors the most common were to give a single value, not always –5, or to use *x* rather than f(*x*). Setting f(*x*) = 0 and solving the quadratic was also not uncommon.

(iii) Far and away the best attempted part, it was good to see so many candidates, even apparently weaker ones, showing an understanding of how to find an inverse. Some good solutions were spoilt by including the negative root. There were a significant minority who, while knowing in principle what they were trying to do, made the error of expanding the quadratic making it impossible to make either *x* or *y* the subject.

(iv) While many understood the significance of being given the line *y* = *x* and produced an attempt at a reflection of their f(*x*), few took into consideration the given domain for f(*x*) and many assumed it to be linear.

(v) This was generally poorly done. While some candidates 'spotted' the answer from their graphs, even incorrect ones, or used trial and error, those who attempted a solution went down the very difficult path of equating the two functions rather than using *y* = *x*. If carried out correctly, this led to a quartic and very little of benefit thereafter.

Answers: **(i)** 1 **(ii)** $f(x) \ge -5$ **(iii)** $1+\sqrt{x+5}$ **(v)** 4

Question 12

It was good to see the last question on the paper attempted by the majority and, in the absence of a choice as in previous years, that so many were able to score marks so late in the paper.

(i) A good knowledge of the factor and remainder theorems was shown by many, although a not uncommon error was to assume that $b = 6$ thus eliminating the need to solve simultaneous equations and losing the attached credit.

(ii) The quadratic factor needed was found correctly by the majority of those who attempted to find it which was in itself the majority of candidates. The most common method was to use long division, a normally difficult skill well demonstrated. There were also few errors solving this quadratic, usually by using the formula, with arithmetic slips occasionally marring an otherwise correct solution.

Answer: **(i)** –14 **(ii)** $-2 \pm \sqrt{6}$

ADDITIONAL MATHEMATICS

Paper 4037/22

Papert 22

Key Messages

In order to do well in this examination, candidates needed to give clear and well thought out answers to questions, with sufficient method being shown to enable marks to be awarded.

General Comments

This examination produced a wide spread of marks, with many scoring highly. Many candidates were well prepared for the examination, giving good and clearly presented answers. In order to improve, other candidates need to understand that their working must be detailed enough to show their method clearly. This is even more important if they make an error. This was highlighted in **Questions 7(ii)** and **9b(ii)**. Showing clear method is also very important if a question starts with the words "Show that…". This indicates that the answer has been given to the candidates and that the marks will be awarded for showing how that answer has been found. The need for this was highlighted in **Question 11(ii)**.

Comments on Specific Questions

Question 1

Most candidates scored at least 2 marks here for obtaining *m* and *c* correctly. Many used $y = 5x - 2$ and stopped at that point thinking they had found the answer. Of those who went on, many scored 4, but a small number square rooted rather than squared for their last step and so lost the last mark. A few candidates stopped when they had found $\sqrt{y} = 5x^2 - 2$. These candidates need to understand that the requirement for giving "*y* in terms of *x*" means that their final answer should be of the form *y* = ….

Answer: $y = (5x^2 - 2)^2$

Question 2

(a) This question was well answered. The need to apply logarithms was appreciated by a good number of candidates. Some omitted brackets when dealing with the power $p + 1$, although the majority of these candidates recovered to give correct working on the next line of their solution. The question demanded an answer accurate to 2 decimal places. Many candidates rounded their working values to 2 decimal places and therefore gave inaccurate final answers. Many rounded to 2 significant figures. Some candidates gave unrounded answers or left their answers in terms of logarithms. A small number of candidates had no evidence of logarithms being used and any answers found using trials were usually only accurate to 1 decimal place and not 2.

(b) The majority of candidates scored at least 2 marks in this question, generally for simplifying the powers of *x* and *y*. The most common errors seen were made when dealing with the brackets or the square root sign, or forgetting that the power had to be applied to all the terms. Some tried to square both the numerator and denominator to deal with the square root in the denominator. Some only attempted to simplify the powers of *x* and *y* or found handling the power of 2 too challenging.

Answers: **(a)** –1.32 **(b)** $2^{\frac{5}{2}}x^6y^{-\frac{1}{2}}$

Question 3

(a) (i) A good number of candidates scored at least one mark in this question, with a fair number scoring both marks. Many wrote down both correct answers and deleted one of them, perhaps thinking that two answers were not required. Occasionally, the correct answer for part **(ii)** was seen in part **(i)** and vice versa.

(ii) Again, a good number scored at least one mark. Often those who scored well in part **(i)** also did so in part **(ii)**. F was a common incorrect answer here.

(b) Many clearly recalled that the inverse function was a reflection in a line, but often the line selected was something other than $y = x$. Candidates need to appreciate that, when given a function on a grid such as this one, with coordinates that can easily be read, swapping the x- and y-coordinates is the simplest way to find the vertices needed for the inverse function. Some candidates were clearly doing this and usually scoring full marks. Some candidates attempted to find the algebraic rule for the function and its inverse and often drew no graph. Some drew graphs that went beyond the grid given.

Answers: **(a)(i)** A, E **(ii)** C, D

Question 4

A fair number of candidates misread the position vector of A as $4\mathbf{i} - 2\mathbf{j}$.

(i) A good number of candidates scored full marks here, with many solutions very well explained and presented. Occasional sign errors were seen. Some candidates confused, for example, $\overrightarrow{OC}$ with $\overrightarrow{CO}$ and, in order to get full marks, notation had to be correct. Some candidates misunderstood the question, thinking they were looking for the vector $\overrightarrow{AC}$ and so stopped working once this had been found.

(ii) Those who found $\overrightarrow{AC}$ in part **(i)** often went on to find $\overrightarrow{OC}$ in this part of the question. Marks were not awarded for part **(i)** here, but candidates were able to gain the 2 marks for finding the correct unit vector. Candidates have a good understanding of unit vectors and how to find them on the whole, with a high proportion scoring 2 marks here for a correct or correct follow through solution. Of those who struggled in this part, it was common for them to be finding a unit fraction rather than a unit vector. Generally this unit fraction was $\dfrac{1}{|\overrightarrow{OC}|}$.

Answers: **(i)** $10\mathbf{i} - 24\mathbf{j}$ **(ii)** $\dfrac{1}{26}(10\mathbf{i} - 24\mathbf{j})$

Question 5

Calculators were not permitted to be used in this question. To score marks here, the length of AX had to be a surd. A small, but not insignificant, number of candidates clearly used calculators to arrive at decimal values for AX, either using Pythagoras' Theorem or trigonometry. Other candidates used the values 7 or 45. Candidates should be aware that a right-angled triangle with hypotenuse of length 7 cannot have a shorter side of length 7 or 45.

Various approaches were seen to this question. The majority started as expected, by forming the area of the trapezium $ABCD$ and equating it to the given area. Rationalising was sometimes incorrectly done, with calculator use again being in evidence in some cases. A good number did rationalise correctly and the majority of these gained full marks. Those not scoring full marks using this approach often made method slips or omitted the length of XD when forming their equation. Those working methodically in small steps usually made fewer errors.

Other approaches seen included subtracting the area of *AXD* from the given area and equating that to the area of trapezium *ABCX*, which was usually successful. Drawing a perpendicular, *Y*, from *B* to *CD*, finding the area of *BYC* and using that to find *YC* and then *XC* was also offered as a solution, again, this was almost always successful.

Some candidates treated *CD* as x in their working, rather than $x + 2$, and subtracted 2 at the end. This was permitted even though it should be discouraged to prevent confusion.

Answer: $4+3\sqrt{5}$

Question 6

(i) The most common error here was to omit justification that the chord *AB* was of length r. Often it was the stronger candidates who omitted this. Candidates need to be clear that any question that asks them to "Show that…." requires clear and full method to be shown to gain full marks. Many used the cosine rule or trigonometry, mostly with success and many spotted the simpler route of justification that triangle *OAB* was equilateral. A small number of candidates scored 0 marks as they were working back from the given statement. Some excellent answers were seen.

(ii) A small number of candidates worked out a correct expression for r but omitted to evaluate it as a decimal. A fair number of candidates found the correct expression and evaluated it incorrectly. An answer of 8.27… from $78 \div 3 \div \pi$ was common among these candidates.

A large number of candidates used the correct method for calculating the area of the segment – either in steps or as one formula. A small number calculated the area of the sector only. It is possible that these candidates confused the words "sector" and "segment". Some candidates, calculating the areas in steps and then subtracting, prematurely rounded their values. They were penalised the final mark for accuracy, if this was the case. Some candidates had their calculator in the wrong mode; others seem to believe that the unit of measure is "π radians" rather than "radians".

Answers: **(ii)** $r = 12.7$ Area of segment = 14.6

Question 7

(i) This part of this question was very well done by the majority of candidates.

(ii) This part was slightly less well done, though still well done by a very good number of candidates. The common error was to differentiate $\sin x$ as $-\cos x$ and then apply the product rule correctly.

(iii) Again, a well done question. Common errors were to differentiate $\tan x$ as $\sec x$ or to omit brackets around the $1+e^{2x}$ term when the quotient rule was being applied. Occasionally the quotient rule was misapplied with the numerator being reversed or the square being omitted from the denominator. Those choosing to use the product rule often made a slip when differentiating $\left(1+e^{2x}\right)^{-1}$.

Answers: **(i)** $-60(3-5x)^{11}$ **(ii)** $x^2\cos x + 2x\sin x$ **(iii)** $\dfrac{\left(1+e^{2x}\right)\sec^2 x - 2e^{2x}\tan x}{\left(1+e^{2x}\right)^2}$

Question 8

(i) Very well done – the majority of candidates scored 2 marks. A common error, generally from those using the $y = mx + c$ approach, was to simplify $6=\frac{1}{2}\times 2+c$ to $c = 6$. Some lost the accuracy mark for leaving their answer in the form $2y = x + 10$, for example.

(ii) Again, very well done, with the majority of candidates scoring 2 marks here.

(iii) The solutions offered here were variable. Strong candidates generally used the expected approach and solved simultaneous equations and arrived at the correct pair of points. These candidates presented clear and tidy solutions which were easy to follow and credit. Weaker candidates wrote down the length equation, often in radical form, and made no real progress beyond that. Commonly the next step was to square root term by term, rather than squaring both sides or separate the relationship into $(x+6)^2 = 100$ and $(y-2)^2 = 100$, for example. The presentation of weaker candidates was often such that their thinking was difficult to follow.

Of those using the simultaneous equations approach: A good number of those who did identify a valid approach realised that a correct strategy was to substitute their equation of *BC* into their length equation, and these candidates were often scoring the two method marks for forming and solving a quadratic equation at this point. There were, however, many method errors/slips made when doing this.

Of those using an inspection approach: Some did find by inspection, either on the length equation or on the equation of *BC*, one or two points that satisfied one of the relationships. The next step, which would be to verify their points in the other relationship, was usually omitted. Therefore the method marks were not earned. Candidates need to be aware that this method was ***only*** reasonable as the solutions were integer values, which generally will not be the case.

Other approaches: A small but significant number used an area approach – often based on the absolute value of the determinant. This was done well and with a high level of success. Others tried vector methods, based on *B* as being the "origin" and using the gradient of *BC* to then work out the correct vector components from *B* to the two positions of *C*. Again this method was done well and usually by very good candidates. A small number assumed that the *x*-coordinate was 4 or the *y*-coordinate of *C* was 2, without any justification for this being given at all. Some were assuming that *AC* was parallel to the *x*-axis without any justification or reasoning for this and lost marks as their method was based on this assumption or was unclear. Others falsely assumed that the *x*- or *y*-coordinate of *C* had to be the same as one of the coordinates of one of the given points. Many geometric arguments were attempted but the reasoning behind them was often poorly explained or not explained at all and these were not credited if key steps in the process were omitted and therefore needed to be assumed.

Answers: **(i)** $y = \frac{1}{2}x + 5$ **(ii)** $y - 6 = -2(x - 2)$ **(iii)** $(0, 10), (4, 2)$

Question 9

(a) A very common error was to treat this as a straight line and to work out the value of *k* using $\frac{14-6}{0+2} = 4$ leading to *c* = 10 or *c* = 14. Incorrect answers were as common as correct answers. Those who found a correct pair of equations most often solved them successfully to gain full marks. There were a small number of manipulation errors seen when handling the equation $6 = \frac{k}{9} + c$, however.

(b) (i) This part was usually well done with only the occasional slip in writing down the answer or some premature approximation resulting in 78.2 for example.

(ii) Many found it beyond them to manipulate the equation once they had substituted $y = 20$. A common error was to "factor out" e^x or e^{-x} in some way and reduce the equation to 2 terms. Another common error was to take logs at this stage, term by term.

Many seemed to be using equation solving functions on their calculators, as the correct decimal answer appeared after incorrect working or no working had been shown. Calculator use such as this should be encouraged only for checking answers and not as a substitute for showing a correct method. These candidates were only credited if there was enough method shown, i.e. if $e^x = 3$ at least was seen, but they could not obtain more than 2 of the 4 marks available.

A good number of candidates did score full marks and they clearly understood that the solution –8 needed to be discarded.

Answers: **(a)** $c = 5$, $k = 9$ **(b)(i)** 79.2 **(ii)** $x = \ln 3$

Question 10

(a) (i) Some excellent sketches were seen for this question, with a high proportion scoring 3 or 4 marks. Occasionally, the shape of the curve was insufficiently accurate to score, with the shape of each cycle being more parabolic than sinusoidal. Some candidates did not sketch the graph over its full domain, stopping at $x = 180°$. Some candidates did not start or finish at the correct point and sketched the rest of the graph correctly. A good number of sketches were very well presented and had been carefully drawn, others were less so.

(ii) This was generally correct. A few answers of 6, this being the common incorrect answer.

(iii) Again, generally correct with the common incorrect answer being 2, the number of cycles.

(b) Many scored 1 mark for writing down $\operatorname{cosec} x = \frac{1}{\sin x}$ or similar. A good number scored 2 marks for also using the relationship $\cos^2 x + \sin^2 x = 1$ correctly in the required way. Only the strongest candidates appreciated that the expression needed to be negative since the angle was in the 4th quadrant and scored full marks.

Answers: **(a)(ii)** 3 **(iii)** 180 **(b)** $-\frac{1}{\sqrt{1 - p^2}}$

Question 11

(i) A common error was $-3(x-4)^{-2}$ for the second term, being unable to subtract 1 from a negative exponent seemingly. Another common error was, after correct work, incorrect simplification to $\frac{1}{(x-4)^4}$ or $-\frac{1}{(x-4)^4}$. A small number of candidates thought they were differentiating a product here, rather unfortunately, as the lengthy work they undertook gained no credit. Candidates must read the question carefully. A small number of candidates used the quotient rule to differentiate the second term, often successfully, but frequent slips were made by candidates using this lengthier method.

(i) Candidates could only score here if they had a correct first derivative in part **(i)**. Some solved the quartic and showed enough evidence of correct work to gain the method mark. Some who attempted to solve the quartic multiplied out or took square roots at an incorrect point and made no further valid progress. A good number correctly verified that when $x = 3$ and when $x = 5$ the first derivative was zero. Very few candidates gave sufficient evidence of their method to show that the *y*-coordinates were as given. Many were simply showing that the *x*-coordinates were as given and then simply stating the given answers. Again, candidates need to be clear that any question that asks them to "Show that…." requires clear and full method to be shown to gain full marks.

(i) The majority of candidates applied the second derivative test and did so very well. A small number of candidates applied the second derivative test correctly but came to the incorrect conclusion. A small number also did not make it explicitly clear which point they were testing. Some simply stated that one point was a maximum and the other a minimum with no method being shown. A very few candidates attempted to use the first derivative test. This method was often spoiled by a selection of points which were insufficiently close to the turning points, resulting in a false conclusion.

(iv) Very well answered, with a very high number of candidates scoring 2 marks for the correct, unsimplified expressions. Some candidates did go on to simplify incorrectly, which affected their answer to part **(v)**.

(v) Again, there were a good number of correct answers. Slips in signs or evaluation resulted in several candidates gaining the method mark only. Very many did substitute the correct limits in the correct order and indicated that they knew that they should subtract one from the other, even if their integrated expression was incorrect. Some provided no working at all except the correct answer. Some gave correct answers following incorrect working. These candidates probably used the numerical integration function on their calculator. Again, calculator use such as this should be encouraged only for checking answers and not as a substitute for showing a correct method.

Answers: **(i)** $\frac{dy}{dx} = 3 - 3(x-4)^{-4}$, $\frac{d^2y}{dx^2} = 12(x-4)^{-5}$ **(iv)** $\frac{3x^2}{2} - \frac{(x-4)^{-2}}{2} + c$ **(v)** 16.875