



**Cambridge International Examinations**  
Cambridge Ordinary Level

---

**ADDITIONAL MATHEMATICS**

**4037/12**

Paper 1

**May/June 2016**

MARK SCHEME

Maximum Mark: 80

---

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

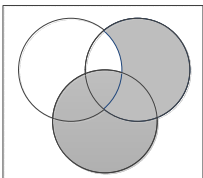
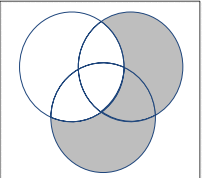
Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE<sup>®</sup>, Cambridge International A and AS Level components and some Cambridge O Level components.

<b>Page 2</b>	<b>Mark Scheme</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>Cambridge O Level – May/June 2016</b>	<b>4037</b>	<b>12</b>

### Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

Question	Answer	Marks	Guidance
<b>1 (a)</b>	$Y \subset X$ or $Y \subseteq X$ only $Y \cap Z = \emptyset$ or $\{ \}$ only	<b>B1</b> <b>B1</b>	
<b>(b)</b>	<b>(i)</b>  <b>(ii)</b> 	<b>B1</b> <b>B1</b>	
<b>2 (i)</b>	$32 - \frac{20}{x} + \frac{5}{x^2}$	<b>B3</b>	<b>B1</b> for each correct term – must be integers
<b>(ii)</b>	$(3 \times 32) + \left( -\frac{20}{x} \times 4x \right) = 16$  Accept $16x^0$	<b>M1</b> <b>A1</b>	for $(3 \times \text{their } 32) + \left( \frac{\text{their } (-20)}{x} \times 4x \right)$
<b>3 (i)</b>	$\mathbf{b - c} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$  $4 + y^2 = 36 + 4$ $y = \pm 6$	<b>B1</b> <b>M1</b> <b>A1</b>	may be implied by further correct working  for one correct attempt at using the modulus
<b>(ii)</b>	$\mu + 4 = 2\lambda$ or $-4\mu + 24 = 7\lambda$ $\mu - 4 = -\lambda$ or $8\mu - 8 = \lambda$  leading to $\mu = \frac{4}{3}$ , $\lambda = \frac{8}{3}$ oe allow 1.33 and 2.67 or better	<b>B1</b> <b>B1</b> <b>DB1</b>	for one correct equation in $\mu$ and $\lambda$ for a second correct equation in $\mu$ and $\lambda$  for both, must have both previous <b>B</b> marks

Page 3	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2016	4037	12

Question	Answer	Marks	Guidance
4	$(4 + \sqrt{5})x^2 + (2 - \sqrt{5})x - 1 = 0$ $x = \frac{-(2 - \sqrt{5}) \pm \sqrt{(2 - \sqrt{5})^2 - 4(4 + \sqrt{5})(-1)}}{2(4 + \sqrt{5})}$ $x = \frac{-(2 - \sqrt{5}) \pm \sqrt{9 - 4\sqrt{5} + 16 + 4\sqrt{5}}}{2(4 + \sqrt{5})}$ $= \frac{-(2 - \sqrt{5}) + 5}{2(4 + \sqrt{5})}$ $= \frac{3 + \sqrt{5}}{2(4 + \sqrt{5})}$ $= \frac{(3 + \sqrt{5})(4 - \sqrt{5})}{2(4 + \sqrt{5})(4 - \sqrt{5})}$ $= \frac{7 + \sqrt{5}}{22}$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>You must be convinced that a calculator is not being used.</b></p> <p>for use of quadratic formula (allow one sign error), allow <math>b^2 = 9 - 4\sqrt{5}</math></p> <p>all correct</p> <p>for attempt to simplify the discriminant (minimum of 4 terms must be seen in discriminant, 2 terms involving <math>\sqrt{5}</math> and 2 constant terms)</p> <p>for <math>\frac{3 + \sqrt{5}}{2(4 + \sqrt{5})}</math> or <math>\frac{3 + \sqrt{5}}{8 + 2\sqrt{5}}</math>, ignore negative solution if included</p> <p>for attempt to rationalise an expression of the form <math>\frac{a \pm b\sqrt{5}}{c \pm d\sqrt{5}}</math> as part of their solution of the quadratic Must obtain an integer denominator</p> <p><b>Final A1 can only be awarded if all previous marks have been obtained</b></p>
5 (i)	$(1 - \cos \theta)(1 + \sec \theta)$ $= 1 - \cos \theta + \frac{1}{\cos \theta} - \frac{\cos \theta}{\cos \theta}$ $= \sec \theta - \cos \theta$ $= \frac{1}{\cos \theta} - \cos \theta$ $= \frac{1 - \cos^2 \theta}{\cos \theta}$ $= \frac{\sin^2 \theta}{\cos \theta}$ $= \sin \theta \tan \theta \quad \text{www}$	<p><b>M1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>M1 for expansion and use of <math>\sec \theta = \frac{1}{\cos \theta}</math> consistently, allow one sign error</p> <p>for attempt at a single fraction, dependent on first M1</p>

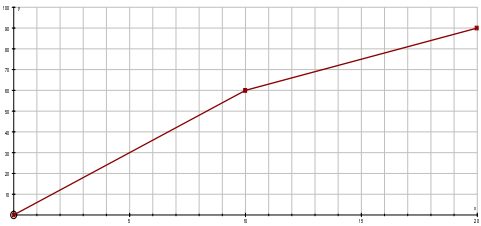
<b>Page 4</b>	<b>Mark Scheme</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>Cambridge O Level – May/June 2016</b>	<b>4037</b>	<b>12</b>

Question	Answer	Marks	Guidance
(ii)	<p>Alternative method:</p> $(1 - \cos \theta) \left( \frac{\cos \theta + 1}{\cos \theta} \right)$ $= \frac{1 - \cos^2 \theta}{\cos \theta}$ $= \frac{\sin^2 \theta}{\cos \theta}$ $= \sin \theta \tan \theta \quad \text{www}$ <p><math>\sin \theta \tan \theta = \sin \theta</math>  <math>\sin \theta (\tan \theta - 1) = 0</math></p> <p><math>\tan \theta = 1, \theta = \frac{\pi}{4},</math> allow 0.785 or better  <math>\sin \theta = 0, \theta = 0, \pi</math> or 3.14 or better</p>	<p><b>M1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>for attempt at a single fraction for second factor and use of <math>\sec \theta = \frac{1}{\cos \theta}</math></p> <p>for expansion</p> <p>for <math>\theta = \frac{\pi}{4}</math> from <math>\tan \theta = 1</math></p> <p>for <math>\theta = 0</math> from <math>\sin \theta = 0</math></p> <p>for <math>\theta = \pi</math> from <math>\sin \theta = 0</math></p>
6	$\frac{d}{dx} \left( e^{3x} (4x+1)^{\frac{1}{2}} \right)$ $= e^{3x} \frac{1}{2} \times 4(4x+1)^{-\frac{1}{2}} + 3e^{3x} (4x+1)^{\frac{1}{2}}$ $= \frac{2e^{3x}}{(4x+1)^{\frac{1}{2}}} + 3e^{3x} (4x+1)^{\frac{1}{2}}$ $= \frac{e^{3x}}{(4x+1)^{\frac{1}{2}}} (2 + 12x + 3)$ $= \frac{e^{3x}}{(4x+1)^{\frac{1}{2}}} (12x + 5)$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p>	<p>for <math>re^{3x} (4x+1)^{-\frac{1}{2}}</math> must be part of a sum, <math>r = \frac{1}{2}</math> or 2 or <math>\frac{1}{2} \times 4</math></p> <p>for <math>se^{3x} (4x+1)^{\frac{1}{2}}</math> must be part of a sum, <math>s</math> is 1 or 3</p> <p>for all correct, allow unsimplified</p> <p>for <math>\frac{e^{3x}}{(4x+1)^{\frac{1}{2}}} (a+bx)</math>, dependent on first 2 <b>B</b> marks, must be using a correct method, collecting terms in the numerator correctly</p>
7 (i)	$\cos 3x = \frac{1}{2}, \quad x = \frac{\pi}{9} \text{ or } 0.349, 20^\circ,$ allow 0.35	<p><b>M1</b></p> <p><b>A1</b></p>	<p>for correct attempt to solve the trigonometric equation</p>
(ii)	$B \left( \frac{\pi}{3}, 3 \right)$ or $(1.05, 3), (60^\circ, 3)$	<p><b>B1B1</b></p>	<p><b>B1</b> for each, must be in correct position or in terms of <math>x =</math> and <math>y =</math></p>

Question	Answer	Marks	Guidance
(iii)	$\int_{\frac{\pi}{9}}^{\frac{\pi}{3}} 1 - 2 \cos 3x \, dx = \left[ x - \frac{2}{3} \sin 3x \right]_{\frac{\pi}{9}}^{\frac{\pi}{3}}$ $= \frac{\pi}{3} - \left( \frac{\pi}{9} - \left( \frac{2}{3} \times \frac{\sqrt{3}}{2} \right) \right)$ $= \frac{2\pi}{9} + \frac{\sqrt{3}}{3} \text{ oe or } 1.28$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p>	<p>for <math>x \pm a \sin 3x</math> attempt to integrate at least one term</p> <p>for correct integration</p> <p>for correct use of limits from (i) and (ii), must be in radians</p>
8 (i)	$\lg y = x^2 \lg b + \lg A$ $\lg b = \pm 0.21$ $b = 0.617$ allow 0.62, $10^{-0.21}$ $\lg A = 0.94$ allow 0.93 to 0.95 $A = 8.71$ allow awrt 8.5 to 8.9  Alternative method  $5.37$ or $10^{0.73} = Ab$ $1.259$ or $10^{0.1} = Ab^4$ $b^3 = 10^{-0.63}$ $b = 0.617$ allow 0.62, $10^{-0.21}$ $A = 8.71$ allow awrt 8.5 to 8.9	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>for <math>\lg b = \pm 0.21</math> may be implied</p> <p>for both equations, allow correct to 2 sf</p>
(ii)	$x = 1.5, x^2 = 2.25$  $y = 2.93$ , allow awrt 2.9 or 3.0	<p><b>M1</b></p> <p><b>A1</b></p>	<p>for correct use of graph <math>y = \text{their}A \times \text{their}b^{1.5^2}</math></p> <p>or <math>\lg y = \lg \text{their}A + (1.5^2 \lg \text{their}b)</math></p>
(iii)	$\lg y = 0.301$ , or $2 = '8.71(0.617)^{x^2}'$  $x = 1.74$ , allow $\sqrt{3}$ or awrt 1.7, 1.8	<p><b>M1</b></p> <p><b>A1</b></p>	<p>for correct use of graph to read off <math>x^2</math></p> <p><math>2 = \text{their}A(\text{their}b)^{x^2}</math> or</p> <p><math>\lg 2 = (\lg \text{their}b)x^2 + \lg(\text{their}A)</math></p>
9 (i)	$y = \frac{2}{3}(3x+10)^{\frac{1}{2}} (+c)$  passes through $\left(2, -\frac{4}{3}\right)$ , so $c = -4$  $y = \frac{2}{3}(3x+10)^{\frac{1}{2}} - 4$ oe	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>for <math>p(3x+10)^{\frac{1}{2}}</math> where <math>p</math> is a constant</p> <p>for <math>\frac{2}{3}(3x+10)^{\frac{1}{2}}</math> oe unsimplified</p> <p>for attempt to find <math>c</math>, must have attempt to integrate, must have the first <b>B1</b></p>

Page 6	Mark Scheme	Syllabus	Paper
	Cambridge O Level – May/June 2016	4037	12

Question	Answer	Marks	Guidance
(ii)	<p>When <math>x = 5</math>,</p> $y = -\frac{2}{3}$ <p>perpendicular gradient = <math>-5</math></p> <p>Equation of normal: <math>y + \frac{2}{3} = -5(x - 5)</math></p> <p>When <math>y = -\frac{5}{3}</math>,</p> $x = 5.2 \text{ oe}$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p>	<p>for attempt at the normal using <i>their</i> perpendicular gradient and <i>their</i> <math>y</math> value (but not <math>y = -\frac{4}{3}</math> or <math>-\frac{5}{3}</math>).</p> <p>for use of <math>y = -\frac{5}{3}</math> in their normal equation to get as far as <math>x = \dots</math></p>
10 (i)	<p>Area: <math>20 = \pi x^2 + xy</math></p> $y = \frac{20 - \pi x^2}{x}$ $P = 2\pi x + 2x + 2y$ $= 2\pi x + 2x + 2\left(\frac{20}{x} - \pi x\right)$ $= 2x + \frac{40}{x}$ <p>Alternative method:</p> $20 = \pi x^2 + xy$ $P = 2\pi x + 2y + 2x$ $= \frac{2}{x}(\pi x^2 + xy) + 2x$ $= \frac{2}{x}(20) + 2x$ $= 2x + \frac{40}{x}$	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>B1</b></p> <p><b>A1</b></p>	<p>for attempt to use perimeter and obtain in terms of <math>x</math> only</p> <p>all steps seen, www AG</p> <p>for attempt to use perimeter and write in <math>\frac{\pi x^2 + xy}{x}</math></p> <p>for replacing <math>\pi x^2 + xy</math> with 20</p> <p>all steps seen, www AG</p>

Question	Answer	Marks	Guidance
(ii)	$\frac{dP}{dx} = 2 - \frac{40}{x^2}$ <p>When <math>\frac{dP}{dx} = 0</math>,</p> $x = 2\sqrt{5} \quad \text{allow } 4.47, \sqrt{20}$ <p>leading to <math>P = 8\sqrt{5}</math>, allow 17.9</p> $\frac{d^2P}{dx^2} = \frac{80}{x^3}$ , always positive so a minimum	<p><b>M1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>for attempt to differentiate</p> <p>for equating to zero and attempt to solve at least as far as <math>x^2 =</math></p> <p>for this statement or use of gradient inspection either side of correct <math>x</math></p>
11 (a) (i)	Distance = area under graph	<b>M1</b>	for attempt to find the area, one correct area seen ( triangle, rectangle or trapezium) as part of a sum.
	= 1275	<b>A1</b>	
(ii)	deceleration is 1.5 oe	<b>B1</b>	
(b)		<b>B1</b>	for a straight line between (0,0) and (10,60)
		<b>B1FT</b>	<b>FT</b> a straight line between (10, 60) and (20, 90), a displacement vector $\begin{pmatrix} 10 \\ 30 \end{pmatrix}$ from <i>their</i> (10, <i>their</i> 60)
(c) (i)	$e^{2t}$ is always positive or oe	<b>B1</b>	
(ii)	$a = 8e^{2t}$ $e^{2t} = \frac{3}{2}$ $t = \frac{1}{2} \ln \frac{3}{2}$ , $\ln \sqrt{\frac{3}{2}}$ or $\frac{1}{2} \ln 1.5$	<b>M1</b>	for attempt to differentiate, must be of the form $pe^{2t}$ , equate to 12 and solve.
		<b>A1</b>	Allow fractions equivalent to $\frac{3}{2}$
(iii)	$s = \left[ 2e^{2t} + 6t \right]_{0.4}^{0.5}$ $= (2e + 3) - (2e^{0.8} + 2.4)$ $= (8.436 - 6.851)$ $= 1.59, \text{ allow } 1.58$	<b>M1</b>	for attempt to integrate to get $qe^{2t} + 6t$
		<b>A1</b>	all correct
		<b>DM1</b>	for correct use of limits or considering distances separately, ignore attempts at $c$
		<b>A1</b>	