Cambridge O Level

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Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2016**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

Question	Answer	Marks	Part Marks
1 (a) (i)	10	**B1**	
(ii)	22	**B1**	
(iii)	4	**B1**	
(b) (i)	$Q \subset R$	**B1**	
(ii)	$P \cap Q = \varnothing$, or { }	**B1**	
2	$a = 1,\ b = -3,\ c = -1$	**B3**	**B1** for each
3	$3y^2 + 5y - 2 = 0$	**B1**, **B1**	**B1** for $5y$ or $5\log_3 x$, **B1** for -2
	$y = \frac{1}{3},\ y = -2$	**M1**	for correct attempt at the solution of *their* quadratic equation
	$x = 3^{\frac{1}{3}},\ x = 3^{-2}$	**M1**	for dealing with one base 3 logarithm correctly
	$x = 1.44,\ x = \frac{1}{9}$	**A1**, **A1**	**A1** for each
4 (i)	$32x^{10} - \frac{80}{3}x^7 + \frac{80}{9}x^4$	**B3**	**B1** for each term, powers of x must be simplified
(ii)	Coefficients needed:		
	$\left(3 \times \textit{their} - \frac{80}{3}\right) + \left(1 \times \textit{their}\ 32\right)$	**M1**	for dealing with 2 terms
	$= -48$	**A1**	Allow **A1** for $-48x^7$

Question	Answer	Marks	Part Marks
5 (i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2(3x+2)}$	**B1**	for correct derivative of log function
	When $x=-\frac{1}{3}$, $y=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2}$	**B1**	for $y=0$
	Equation of normal: $y=-\frac{2}{3}\left(x+\frac{1}{3}\right)$	**M1** **A1**	**M1** for attempt at a gradient of a perpendicular from differentiation and the equation of the normal
(ii)	$Q\left(0,-\frac{2}{9}\right)$ or $(0,0.22)$ or better	**B1 ft**	Follow through on *their c* from part (i)
	$R\left(0,\frac{1}{2}\ln 2\right)$ or $(0,0.35)$ or better	**B1**	
	Area of PQR $=\frac{1}{2}\left(\frac{1}{2}\ln 2+\frac{2}{9}\right)\times\frac{1}{3}$ $=0.0948$	**B1**	Allow 0.095
6 (a)	**YX**, **XZ**	**B2**	**B2** for both with no extras **B1** for 1 correct with or without extras **B1** for both correct with extras **B0** for anything else
(b) (i)	$\frac{1}{18}\begin{pmatrix}7 & 1\\ -4 & 2\end{pmatrix}$	**B1**, **B1**	**B1** for $\frac{1}{18}$, **B1** for $\begin{pmatrix}7 & 1\\ -4 & 2\end{pmatrix}$
(ii)	$\mathbf{C}=\mathbf{A}^{-1}\mathbf{B}$ $=\frac{1}{18}\begin{pmatrix}7 & 1\\ -4 & 2\end{pmatrix}\begin{pmatrix}-4 & 2\\ 10 & 4\end{pmatrix}$	**M1**	for pre-multiplication
	$=\begin{pmatrix}-1 & 1\\ 2 & 0\end{pmatrix}$	**A1**, **A1**	**A1** for any correct pair of elements, but must be from correct matrices

Question	Answer	Marks	Part Marks
7 (i)	$(0, \sqrt{3})$ or $(0, 1.73)$ or better	**B1**	
(ii)	$\left(\frac{\pi}{6}, 2\right)$ or $(0.524, 2)$ or better	**B1**, **B1**	**B1** for each
(iii)	$\cos\left(x - \frac{\pi}{6}\right) = 0$	**M1**	for correct attempt to solve trigonometric equation
	$x = \frac{2\pi}{3}$ oe or 2.09 or better	**A1**	
(iv)	$2\sin\left(x - \frac{\pi}{6}\right)$ (+c)	**B1**	
(v)	Area $= \left[2\sin\left(x - \frac{\pi}{6}\right)\right]_0^{\frac{2\pi}{3}}$ $= 2 + 1$ $= 3$	**M1** **A1**	for correct use of **their** limits, in radians, into $k\sin\left(x - \frac{\pi}{6}\right)$.
8 (i)	$47 - 24 = 12\theta$		
	$\theta = \frac{23}{12}$, so $\theta = 1.917$ or better	**M1**	for complete correct method to get $\theta =$
	$\theta = 1.92$ to 2dp	**A1**	must have evidence of working to more than 2 dp, allow if 1.916 seen (truncated)
(ii)	$\sin\frac{\theta}{2} = \frac{CD/2}{12}$	**M1**	for a complete method, may use cosine rule to get *CD*
	CD = awrt 19.6 or 19.7	**A1**	
(iii)	Area of sector = awrt 138	**B1**	for sector area, allow unsimplified
	Area of triangle *AOB* = awrt 67 or 68	**M1**	for a correct attempt at area
	Area of segment = awrt 70 or 71	**M1**	for segment area (*their* sector area – *their* triangle area)
	$AD \times AB$ + segment area = 425	**M1**	for complete method to find *AD*
	leading to *AD* = awrt 18.1 or 18.0	**A1**	Allow **A1** for 18
	Alternative method:		
	Area of sector = awrt 138	**B1**	for sector area
	Difference in length between *BC* (or *AD*) and *OM* where *M* is the midpoint of *CD* = 6.88, allow awrt 6.9	**M1**	for attempt to find difference between parallel sides
	Remaining area consists of two trapezia each of width 9.85 and each of area 143.4 $\frac{1}{2}(2BC - 6.88) \times 9.85 = 143.4$ oe	**M1**	for area of one trapezium $\frac{1}{2}(2BC - \textit{their } 6.88) \times \textit{their } 9.85$ oe
	leading to *AD* = awrt 18.1 or 18.0	**M1** **A1**	for attempt to find either *BC* or *AD*

Question	Answer	Marks	Part Marks
9 (i)	$\mathrm{p}\left(\frac{3}{2}\right)$: $\frac{27a}{8}-\left(4\times\frac{9}{4}\right)+\frac{3b}{2}+18\ \ (=0)$	**M1**	for attempt at $\mathrm{p}\left(\frac{3}{2}\right)$
	$\mathrm{p}'\left(\frac{3}{2}\right)=\left(3a\times\frac{9}{4}\right)-\left(8\times\frac{3}{2}\right)+b\ \ (=0)$	**M1**	for differentiation and attempt at $\mathrm{p}'\left(\frac{3}{2}\right)$
	leading to $9a+4b+24=0$ oe and $27a+4b-48=0$ oe	**M1**	for solution of simultaneous equations, to get either a or b
	leading to $a=4,\ b=-15$	**A1**	for both
(ii)	$(x+2)(2x-3)^2$ oe	**M1**, **A1**	**M1** for attempt at long division or factorisation
(iii)	$(x+2)(2x-3)^2=x+2$ $x+2=0,\ x=-2$	**B1**	Must be using $(x+2)$ correctly using part (ii) to get $x=-2$
	$(2x-3)^2=1$ leading to $x=1,\ x=2$	**M1** **A1**	for solution of the quadratic equation
10 (a) (i)	$20U+\frac{1}{2}\left(U+\frac{U}{2}\right)10=165$	**M1**	for realising that area under the graph is needed and attempt to find an area
		DM1	for equating their area to 165 and attempt to solve
	leading to $U=6$	**A1**	
(ii)	Gradient of line: -0.3	**M1**, **A1**	**M1** for use of the gradient, must be negative
(b) (i)	27	**B1**	
(ii)	$t^2=8\ln 4$ $t=3.33$ or better	**M1** **A1**	for a correct attempt to solve $\mathrm{e}^{\frac{t^2}{8}}=4$
(iii)	acceleration $=3\frac{2t}{8}\mathrm{e}^{\frac{t^2}{8}}\left(\mathrm{e}^{\frac{t^2}{8}}-4\right)^2$	**M1**, **A1**	**M1** for a correct attempt to differentiate using the chain rule
	When $t=1,\ a=6.98$	**M1**, **A1**	**M1**for use of $t=1$ in their acceleration

Question	Answer	Marks	Part Marks
11 (i)	$\ln y = \ln A + x \ln b$	**B1**	may be implied, if equation not seen specifically, by correct values for A and b
	Gradient: $\ln b = -\dfrac{0.12}{8}$, $= -0.015$	**M1**	for use of gradient to obtain ln b
	$b = 0.985$	**A1**	Allow **A1** for $e^{-0.015}$
	Intercept: $\ln A = 0.26$	**DM1**	for use of one of the given points correctly
	$A = 1.30$	**A1**	Allow **A1** for $e^{0.26}$ or 1.3
	Alternative 1		
	$\ln y = \ln A + x \ln b$	**B1**	
	$0.2 = 4 \ln b + \ln A$	**M1**	for one correct equation
	$0.08 = 12 \ln b + \ln A$	**DM1**	for attempt to obtain either ln$A$ or ln$b$ from simultaneous equations
	$A = 1.30$ and $b = 0.985$	**A1**, **A1**	Allow **A1** for $b = e^{-0.015}$ and $a = e^{0.26}$ or 1.3
	Alternative 2		
	$1.22 = Ab^4$	**B1**	
	$1.08 = Ab^{12}$	**B1**	
		M1	for correct attempt to obtain b or A, must already have **B2**
	$A = 1.30$ and $b = 0.985$	**A1**, **A1**	Allow **A1** for $b = e^{-0.015}$ and $a = e^{0.26}$ or 1.3
(ii)	When $x = 6$, $\ln y = 0.17$	**M1**	for $\ln y = \textit{their} \ln A + 6\ \textit{their} \ln b$ or $y = \textit{their}\ A \times (\textit{their}\ b)^6$
	$y = 1.19$	**A1**	allow awrt 1.18 to 1.20
(iii)	When $y = 1.1$, $\ln y = 0.095$	**M1**	for $\ln 1.1 = \textit{their} \ln A + x\ \textit{their} \ln b$ or $1.1 = \textit{their} A \times (\textit{their} b)^x$
	$x = 11$	**A1**	allow 10.5 to 11.5