Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/23**

Paper 2 **October/November 2016**

MARK SCHEME

Maximum Mark: 80

Published

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Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied
www without wrong working

Question	Answer	Mark	Part Marks
1	$\frac{(\sqrt{5}+3\sqrt{3})}{(\sqrt{5}+\sqrt{3})}\times\frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})}$	M1	rationalise with $(\sqrt{5}-\sqrt{3})$
	$=\frac{5+3\sqrt{15}-\sqrt{15}-9}{5-3}$	A1	numerator (3 or 4 terms)
	$=\frac{2\sqrt{15}-4}{2}=\sqrt{15}-2$	A1	denominator and completion
2	$\ln e^{3x}=\ln 6e^{x}$		
	$3x=\ln 6e^{x}$	M1	one law of indices/logs
	$3x=\ln 6+\ln e^{x}$	M1	second law of indices/logs
	$3x=\ln 6+x$		
	$x=\frac{1}{2}\ln 6$ or $\ln\sqrt{6}$ or 0.896	A1	www oe in base 10
3 (i)	$\frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right)=\frac{(1+\cos x)\cos x+\sin x\sin x}{(1+\cos x)^2}$	M1	Quotient Rule (or Product Rule from $(\sin x)(1+\cos x)^{-1}$)
		A1	correct unsimplified
	$=\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$	B1	use of $\sin^2 x+\cos^2 x=1$ oe
	$=\frac{1+\cos x}{(1+\cos x)^2}$	A1	completion
(ii)	$\int_0^2\left(\frac{1}{1+\cos x}\right)dx=\left[\frac{\sin x}{1+\cos x}\right]_0^2$	M1	correct integrand
	awrt 1.56	A1	

Question	Answer	Mark	Part Marks
4 (i)	$\mathrm{p}(2)=0 \to 8+4a+2b-24=0$ $\to (4a+2b=16)$	B1	
	$\mathrm{p}(1)=-20 \to 1+a+b-24=-20$ $\to (a+b=3)$	B1	
		M1	solve *their* linear equations for a or b
	$a=5$ and $b=-2$	A1	
(ii)	$\mathrm{p}(x)=x^3+5x^2-2x-24$	M1	find quadratic factor
	$=(x-2)(x^2+7x+12)$	A1	correct quadratic factor soi
	$=(x-2)(x+3)(x+4)$	M1	factorise quadratic factor and write as product of 3 linear factors
	$\mathrm{p}(x)=0 \to x=2, -3, -4.$	A1	if 0 scored, **SC2** for roots only
5 (i)	$AB^2=\left(\sqrt{3}+1\right)^2+\left(\sqrt{3}-1\right)^2$ $-2\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\cos 60$	M1	use cosine rule
	$=3+1+2\sqrt{3}+3+1-2\sqrt{3}-2$	A1	at least 7 terms
	$=6$	A1	correct completion AG
(ii)	$\dfrac{\sin A}{\sqrt{3}-1}=\dfrac{\sin 60}{\sqrt{6}}$	M1	sine rule (or cosine rule)
	$\sin A=\dfrac{\left(\sqrt{3}-1\right)\sin 60}{\sqrt{6}}=\dfrac{\sqrt{6}-\sqrt{2}}{4}$ oe or 0.259 or 0.2588…	A1	correct explicit expression for $\sin A$ AG
(iii)	Area = $\dfrac{1}{2}\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\sin 60$	M1	correct substitution into $\dfrac{1}{2}ab\sin C$
	$=\dfrac{\sqrt{3}}{2}$	A1	
6 (i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sec^2 x$	B1	
	$x=\dfrac{\pi}{4} \to \dfrac{\mathrm{d}y}{\mathrm{d}x}=\sec^2\dfrac{\pi}{4}=2$	B1	evaluated
	$y=8$	B1	
	Equation of tangent $\dfrac{y-8}{x-\dfrac{\pi}{4}}=2$ $(4-2y=\pi-16,\ y=2x+6.429\ldots,$ $\dfrac{\pi}{4}=0.7853\ldots)$	B1	

Question	Answer	Mark	Part Marks
(ii)	$\sec^2 x = \tan x + 7$ $\tan^2 x - \tan x - 6 = 0$ oe $(\tan x - 3)(\tan x + 2) = 0$ $\tan x = 3$ or $\tan x = -2$ $x = 1.25, \quad 2.03$	M1 M1 A1A1	use $\sec^2 x = 1 + \tan^2 x$ to obtain a 3 term quadratic in tan$x$ solve three term quadratic for tan$x$ extras in range lose final **A1**
7 (i)	$r^2 + h^2 = (0.5h + 2)^2$ oe $r^2 = 0.25h^2 + 2h + 4 - h^2$ $r^2 = 2h + 4 - 0.75h^2$	M1 A1	 correct expansion and r^2 subject and completion www AG
(ii)	$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\left(2h^2 + 4h - 0.75h^3\right)$ $\frac{dV}{dh} = \frac{\pi}{3}\left(4h + 4 - 2.25h^2\right)$ $\frac{dv}{dh} = 0 \to 2.25h^2 - 4h - 4 = 0$ $h = 2.49$ only	B1 M1 A1 M1 A1	any correct form in terms of h only differentiate V correct differentiation equate to 0 and solve 3 term quadratic cao
(iii)	$\frac{d^2V}{dh^2} = \frac{\pi}{3}(4 - 4.5h)$ when $h = 2.49$ (–7.545…) < 0 so maximum	M1 A1	differentiate *their* 3 term $\frac{dV}{dh}$ and substitute *their* h draw correct conclusion www
8 (i)	$\cos TOA = \frac{6}{10} \to$ $TOA = 0.927$	M1 A1	any method
(ii)	area of major sector = $\frac{1}{2}6^2\left(2\pi - 2 \times \textit{their}\,0.927\right)$ (= 79.7) area of half kite = $\frac{1}{2}(6)\sqrt{10^2 - 6^2}$ (=24) area of kite × 2 (=48) complete correct plan awrt 128	 M2 M1 DM1 DM1 A1	 or **M1** for $\frac{1}{2}$ $6^2(2 \times \textit{their}\ 0.927)$ **DM1** for $\pi \times 6^2 - \frac{1}{2}$ $6^2(2 \times \textit{their}\ 0.927)$ any method *their* major sector + *their* kite
(iii)	arc length = $6 \times (2\pi - 2 \times \textit{their}\,0.927) + 2 \times \sqrt{10^2 - 6^2}$) awrt 42.6	 M1 A1	 complete correct method

Question	Answer	Mark	Part Marks
9 (i)	$p = 4$	B1	
(ii)	$\tan\alpha = \pm\frac{1}{3}$ or ±3 or 18.4° or 71.6° seen	M1	could use cos or sin
	108	A1	
(iii)	$\mathbf{r}_A = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} \textit{their } p \\ -3 \end{pmatrix}$	B1	
(iv)	$\mathbf{r}_B = \begin{pmatrix} q \\ -15 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	B1	
(v)	$5 - 3t = -15 - t$	M1	$\mathbf{r}_A = \mathbf{r}_B$ and equate y/**j** and solve for t
	$\to t = 10$	A1	
(vi)	$\begin{pmatrix} 41 \\ -25 \end{pmatrix}$ only	B1	
(vii)	$q = 11$ only	B1	
10 (i)	$\text{fg}(x) = \ln(2\text{e}^x + 3) + 2$	B1	isw
(ii)	$\text{ff}(x) = \ln(\ln x + 2) + 2$	B1	isw
(iii)	$x = 2\text{e}^y + 3$	M1	change x and y and make e^y the subject
	$\text{e}^y = \frac{x-3}{2}$		
	$\text{g}^{-1}(x) = \ln\left(\frac{x-3}{2}\right)$ oe	A1	
(iv)	e^2 or 7.39	B1	
(v)	$\text{gf}(x) = 2\text{e}^{(\ln x + 2)} + 3 = 20$	B1	gf correct and equation set up correctly
	$2\text{e}^{\ln x}\text{e}^2 + 3 = 20$	M1	one law of indices/logs
	$2x\text{e}^2 = 17$	M1	second law of indices/logs
	$x = \frac{17}{2\text{e}^2}$ or 1.15	A1	
			www if 0 scored, **SC2** for 17.3…

Question	Answer	Mark	Part Marks
11 (i)	$\mathbf{A}^2 = \begin{pmatrix} 2 & q \\ p & 3 \end{pmatrix}\begin{pmatrix} 2 & q \\ p & 3 \end{pmatrix} = \begin{pmatrix} 4+pq & 2q+3q \\ 2p+3p & pq+9 \end{pmatrix}$	**B2,1,0**	–1 each error
	$\mathbf{A}^2 - 5\mathbf{A} = 2\mathbf{I} \to 4 + pq - 10 = 2$ or $9 + pq - 15 = 2$	**M1**	equate top left or bottom right elements
	$\to pq = 8$	**A1**	accept $p = \frac{8}{q}$, $q = \frac{8}{p}$
(ii)	$\det \mathbf{A} = 6 - pq$	**B1**	
	$6 - pq = -3p$ and solve	**M1**	*their* det $\mathbf{A} = -3p$ and use *their* $pq = k$ oe to solve for p or q
	$\to p = \frac{2}{3}$	**A1**	
	$q = 12$	**A1**	**FT** from *their* $pq = k$