



Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS

4037/13

Paper 1

October/November 2016

MARK SCHEME

Maximum Mark: 80

Published

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Abbreviations

awrt answers which round to
 cao correct answer only
 dep dependent
 FT follow through after error
 isw ignore subsequent working
 oe or equivalent
 rot rounded or truncated
 SC Special Case
 soi seen or implied
 www without wrong working

Question	Answer	Marks	Part Marks
1		<p>B1</p> <p>B1</p> <p>B1</p>	<p>for symmetrical shape as in the diagram with curved maxima of equal height and cusps on the x-axis</p> <p>for a complete ‘curve’ with all low points on the x-axis and all high points on $y = 2$</p> <p>for a complete ‘curve’ meeting the x-axis at $x = 30^\circ, 90^\circ, 150^\circ$ only.</p>
2	$= \frac{4m^2 - 9}{2m + 3}$ $= \frac{(2m - 3)(2m + 3)}{2m + 3}$ $= 2m - 3$ <p>Alternative Method</p> $(4m\sqrt{m} - \frac{9}{\sqrt{m}})$ $= (2\sqrt{m} + \frac{3}{\sqrt{m}})(Am + B)$ <p>Comparing coefficients $2A = 4, 3A + 2B = 0, 3B = -9$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>for multiplying each term by \sqrt{m}, using a common denominator of \sqrt{m} or for multiplying numerator and denominator by $2\sqrt{m} - \frac{3}{\sqrt{m}}$</p> <p>for a correct expression that will cancel $\frac{(2m - 3)(2m + 3)}{2m + 3}, \frac{(4m^2 - 9)(2m - 3)}{(4m^2 - 9)}$ $\frac{(2m - 3)(2m + 3)(2m - 3)}{(2m + 3)(2m - 3)}$, or equivalents</p> <p>for $2m - 3$ or $A = 2, B = -3$</p> <p>for correct expansion</p> <p>for correct comparisons to obtain A and B for $2m - 3$ or $A = 2, B = -3$</p>

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Question	Answer	Marks	Part Marks
3 (i)	$3x^2 - 2xp + (p+3) = 0$ $(-2p)^2 - 4 \times 3 \times (p+3) \geq 0$ oe $p^2 \geq 3(p+3)$ or $4p^2 - 12p - 36 \geq 0$ $p^2 - 3p - 9 \geq 0$	M1	for obtaining a 3-term quadratic in the form $ax^2 + bx + c (= 0)$
		DM1	for correct substitution of <i>their</i> a , b and c into ' $b^2 - 4ac$ ' and use of discriminant.
		A1	for full correct working, \geq the only sign used, \geq used before division by 4 and \geq used in answer line and penultimate line.
(ii)	Correct method of solution $p^2 - 3p - 9 = 0$ leading to critical values $p = \frac{3 \pm 3\sqrt{5}}{2}$ $p \leq \frac{3 - 3\sqrt{5}}{2}$, $p \geq \frac{3 + 3\sqrt{5}}{2}$	M1	for correct substitution in the quadratic formula or for correct attempt to complete the square. (allow 1 sign error in either method)
		A1	for both correct critical values
		A1	for correct range
4 (i)	$64 - 48x + 15x^2$	B3	for each correct term
(ii)	$(4 \times '64') + (2 \times '-48') + (3 \times '15')$ $= 205$ cao	M1	for correctly obtaining three products using <i>their</i> coefficients in (i)
		A1	for two correct out of three products (unsimplified) cao
		A1	for 205 selected as final answer
5 (i)	$\log_9 xy = \log_9 x + \log_9 y$ $= \frac{\log_3 x}{\log_3 9} + \frac{\log_3 y}{\log_3 9}$ $= \frac{\log_3 x}{2} + \frac{\log_3 y}{2} = \frac{5}{2}$ $\log_3 x + \log_3 y = 5$ Alternative method $\log_9 xy = \frac{5}{2}$ $xy = 9^{\frac{5}{2}} = 3^5$ $\log_3 xy = 5$ $\log_3 x + \log_3 y = 5$	M1	for use of $\log AB = \log A + \log B$
		M1	for correct method for change of base. Division by $\log_3 9$ should be seen and not implied.
		A1	for dealing with 2 correctly and 'finishing off'
		M1	for obtaining xy as a power of 3
		M1	for correct use of \log_3
		A1	for using law for logs and arriving at correct answer

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Question	Answer	Marks	Part Marks
(ii)	$\log_3 x(5 - \log_3 x) = -6$ $-(\log_3 x)^2 + 5\log_3 x = -6$ $(\log_3 x)^2 - 5\log_3 x - 6 = 0$ leading to $\log_3 x = 6, \log_3 x = -1$ $x = 729, x = \frac{1}{3}$ $y = \frac{1}{3}, y = 729$	M1 A1 A1 DM1 A1	for substitution, correct expansion of brackets and manipulation to get a 3 term quadratic for a correct quadratic equation in the form $ax^2 + bx + c = 0$ for both solutions for method of solution of $\log_3 x = k$ or $\log_3 y = k$ for all x and y correct
6 (i)	$\frac{6x}{3x^2 - 11}$	M1 A1	M1 for $\frac{mx}{3x^2 - 11}$
(ii)	$p = \frac{1}{6}$	B1	FT for $p = \frac{1}{m}$
(iii)	$\frac{1}{6}\ln(3a^2 - 11) - \frac{1}{6}\ln 1 = \ln 2$ $\ln(3a^2 - 11) = \ln 2^6$ $3a^2 - 11 = 64$ $a = 5$ only	M1 DM1 DM1 A1	for correct use of limits in $p \ln(3x^2 - 11)$ May be implied by following equation for dealing with logs correctly for solution of $3a^2 - 11 = k$ for 5 obtained from an exact method

Question	Answer	Marks	Part Marks
7 (i)	$\ln y = \ln A + \frac{b}{x}$ Gradient: $b = -0.8$ Intercept or use of equation: $\ln A = 4.7$ $A = 110$ Alternative Method $3.5 = \ln A + 1.5b$ and $1.5 = \ln A + 4b$ leading to $b = -0.8$ $\ln A = 4.7$ and $A = 110$ Alternative Method $e^{1.5} = Ae^{4b}$ $e^{3.5} = Ae^{1.5b}$ leading to $b = -0.8$ and $A = 110$	B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1 B1	for equation, may be implied, must be using \ln unless recovered for $b = -0.8$ oe for $\ln A = 4.7$ oe, allow 4.65 to 4.75 for $A = 110$, allow 105 to 116 Allow A in terms of e for one equation for $b = -0.8$ for $\ln A = 4.7$ for $A = 110$ or $e^{4.7}$ for $e^{1.5} = Ae^{4b}$ or $4.48 = Ae^{4b}$ for $e^{3.5} = Ae^{1.5b}$ or $33.1 = Ae^{1.5b}$ for $b = -0.8$ for $A = 110$ or $e^{4.7}$
(ii)	When $x = 0.32$, $\frac{1}{x} = 3.125$, $\ln y = 2.2$ $y = 9$ (allow 8.5 to 9.5) or $e^{2.2}$	M1 A1	for a complete method to obtain y , using either the graph, using <i>their</i> values in the equation for $\ln y$ or using <i>their</i> values in the equation for y .
(iii)	When $y = 20$, $\ln y = 3$, $\frac{1}{x} = 2.125$ so $x = 0.47$ (allow 0.45 to 0.49)	M1 A1	for a complete method to obtain x , using either the graph, using <i>their</i> values in the equation for $\ln y$ or using <i>their</i> values in the equation for y .

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Question	Answer	Marks	Part Marks
8 (a) (i)	$\frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \sin \theta} = \frac{\frac{1}{\sin \theta}}{\frac{1}{\sin \theta} - \sin \theta}$ $= \frac{1}{1 - \sin^2 \theta} \text{ or } = \frac{\frac{1}{\sin \theta}}{\frac{(1 - \sin^2 \theta)}{\sin \theta}}$ $= \frac{1}{\cos^2 \theta}$ $= \sec^2 \theta$ <p>Alternative Method using cosec</p> $\frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \sin \theta} = \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - \frac{1}{\operatorname{cosec} \theta}}$ $= \frac{\operatorname{cosec}^2 \theta}{\operatorname{cosec}^2 \theta - 1}$ $= \frac{1 + \cot^2 \theta}{\cot^2 \theta}$ $= \tan^2 \theta + 1 = \sec^2 \theta$	<p>M1 for using $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and either attempt to multiply top and bottom by $\sin \theta$ or an attempt to combine terms in denominator.</p> <p>DM1 for correct use of $1 - \sin^2 \theta = \cos^2 \theta$</p> <p>A1 for completing the proof</p>	
	(ii)	$\cos^2 \theta = \frac{1}{4}, \quad \cos \theta = \pm \frac{1}{2}$ $\text{or } \tan^2 \theta = 3, \quad \tan \theta = \pm \sqrt{3}$ $\text{or } \sin^2 \theta = \frac{3}{4}, \quad \sin \theta = \pm \frac{\sqrt{3}}{2}$ $\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$	
(b)	$\tan\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$ $x = \frac{\pi}{6} - \frac{\pi}{4}, \frac{7\pi}{6} - \frac{\pi}{4}, \frac{13\pi}{6} - \frac{\pi}{4}$ $x = \left(-\frac{\pi}{12}\right), \frac{11\pi}{12}, \frac{23\pi}{12}$	<p>M1 for correct order of operations, can be implied by $x = -\frac{\pi}{12}$</p> <p>A1,A1 A1 for $x = \frac{11\pi}{12}$</p> <p>A1 for $x = \frac{23\pi}{12}$</p> <p>If there are extra solutions in range in addition to the two correct ones then A1A0</p>	

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Question	Answer	Marks	Part Marks
9	(a) (i) ${}^{18}C_5 = 8568$ mmm	B1	
	(ii) Either ${}^{10}C_4 \times {}^8C_1 = 1680$ ${}^{10}C_3 \times {}^8C_2 = 3360$ ${}^{10}C_2 \times {}^8C_3 = 2520$ ${}^{10}C_1 \times {}^8C_4 = 700$ Total = 8260	B1 B2,1,0	for a correct plan B2 4 correct numbers with no extras B1 3 correct numbers (out of 3 or 4)
	Or <i>their</i> ${}^{18}C_5 - ({}^{10}C_5 + {}^8C_5)$ $8568 - (252 + 56)$ Total = 8260	B1 B1 B1 B1	for correct plan for 252 subtracted for 56 subtracted for correct total
	(b) (i) ${}^{10}P_6 = 151200$	B1	
	(ii) $4 \times {}^8P_4 \times 3$ = 20160	M1 A1	for correct unsimplified for correct numerical answer
	(iii) Answer to (i) - 7P_6 = 146160	M1 A1 A1	for correct plan for correct unsimplified for correct numerical answer
	Alternative: 1 symbol: 45360 2 symbols: 75600 3 symbols: 25200 Total: 146160	B2,1,0 B1	B2 for all 3 correct B1 for 2 correct (out of 2 or 3) for correct sum

Question	Answer	Marks	Part Marks
10 (i)	$f(x) = 3x^2 - 4e^{2x} (+c)$ passing through $(0, -3)$ $-3 = 3 \times 0 - 4e^0 + c$ $f(x) = 3x^2 - 4e^{2x} + 1$	M1 A1 A1 DM1	for one correct term for one correct term $3x^2$ or $-4e^{2x}$ for a second correct term with no extras for correct method to find c .
(ii)	$f'(0) = -8$ Normal: $y + 3 = \frac{1}{8}x$ $8y + 24 = x$ $y = 2 - 3x$ leads to $x = \frac{8}{5}$ oe $\text{Area} = \frac{1}{2} \times 3 \times \frac{8}{5} = 2.4$ oe	B1 M1 DM1 A1 B1	for $m = \frac{1}{8}$ for equation of normal using $m = \frac{1}{8}$ for solving normal equation simultaneously with $y = 2 - 3x$ to get a value of x for $x = \frac{8}{5}$, 1.6 oe FT for a numerical answer equal to $\left \frac{1}{2} \times 3 \times \text{their } x \right $
11 (i)	$a = 8t - 8$ When $t = 3$, $a = 16$	B1 B1	for $8t - 8$ for 16
(ii)	0.5, 1.5	B1, B1	B1 for each
(iii)	$s = \frac{4}{3}t^3 - 4t^2 + 3t$ when $t = \frac{1}{2}$, $s = \frac{2}{3}$ when $t = \frac{3}{2}$, $s = 0$ total distance travelled = $\frac{4}{3}$	M1 A1 DM1 DM1 A1	for at least two terms correct all correct for calculating displacement when either $t = \frac{1}{2}$ or $t = \frac{3}{2}$ for calculating displacement at $t = \frac{1}{2}$ and doubling. for $\frac{4}{3}$ oe allow 1.33
	Alternative method	M1A1 DM1 DM1 A1	As before DM1 for calculating displacement when $t = 0.5$ or for calculating distance travelled between $t = 0.5$ and $t = 1.5$ DM1 for doubling distance travelled between $t = 0.5$ and $t = 1.5$ or for adding that distance to displacement at $t = 0.5$ A1 for $\frac{4}{3}$ oe allow 1.33