Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/23**

Paper 2 **October/November 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

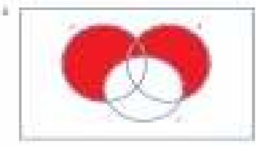
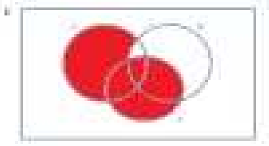
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1(a)	 	B2	**B1** for each
1(b)	$\mathrm{n}(P')$ $= 18$	B1	
	$\mathrm{n}((Q \cup R) \cap P)$ $= 11$	B1	
	$\mathrm{n}(Q' \cup P)$ $= 29$	B1	
2	$3x - 1 = 5 + x$ $x = 3$	B1	
	$3x - 1 = -5 - x$ oe	M1	M1 not earned if incorrect equation(s) present
	$x = -1$	A1	
3	$\dfrac{p(\sqrt{3}+1)+(\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = q + 3\sqrt{3}$	M1	on LHS take common denominator or rationalise each term or multiply throughout
	$p(\sqrt{3}+1)+(\sqrt{3}-1) = 2q + 6\sqrt{3}$ oe	A1	correct eqn with no surds in denominators of LHS
	equate surd/non surd parts	M1	equate and solve for p or q ($\neq 0$)
	$p = 5$ and $q = 2$	A1	
4	$\log_3 3 = 1$ or $\log_3 9 = 2$	B1	implied by one correct equation
	$x + 1 = 3y$	B1	
	$x - y = 9$	B1	
	solve correct equations for x or y	M1	
	$x = 14$ and $y = 5$	A1	
5(i)	$\overrightarrow{OX} = \lambda(1.5\mathbf{b} + 3\mathbf{a})$	B1	
5(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ or $\overrightarrow{BA} = \mathbf{a} - \mathbf{b}$	B1	
	$\overrightarrow{OX} = \mathbf{a} + \mu(\mathbf{b} - \mathbf{a})$	B1	
5(iii)	$1.5\lambda = \mu$ or $3\lambda = 1 - \mu$	M1	$\overrightarrow{OX} = \overrightarrow{OX}$ and equate for **a** or **b**
	$\mu = \dfrac{1}{3}$ $\lambda = \dfrac{2}{9}$	A2	**A1** for each

Question	Answer	Marks	Guidance
5(iv)	$\frac{AX}{XB}=\frac{1}{2}$	B1	Accept 1 : 2 but not $\frac{1}{2}:1$
5(v)	$\frac{OX}{XD}=\frac{2}{7}$	B1	Accept 2 : 7 but not $\frac{2}{7}:1$
6(i)	$f^2 = f(f)$ used algebraic $([(x+2)^2+1]+2)^2+1$	M1	numerical or algebraic
	17	A1	
6(ii)	$x=\frac{y-2}{2y-1}$	M1	change x and y
	$2xy-x=y-2 \to y(2x-1)=x-2$	M1	**M1dep** multiply, collect y terms, factorise
	$y=\frac{x-2}{2x-1}$ $[=g(x)]$	A1	correct completion
6(iii)	$\mathrm{gf}(x)=\frac{\left[(x+2)^2+1\right]-2}{2\left[(x+2)^2+1\right]-1}$ oe	B1	
	$\frac{(x+2)^2-1}{2(x+2)^2+1}=\frac{8}{19}$ $3(x+2)^2=27$ oe $3x^2+12x-15=0$	M1	*their* gf $=\frac{8}{19}$ and simplify to quadratic equation
	solve quadratic	M1	**M1dep** Must be of equivalent form
	$x=1$ $x=-5$	A1	
7(i)	$v=0\to\cos 2t=\frac{1}{3}$	M1	set $v=0$ and solve for $\cos 2t$
	$\to t=0.615$ or 0.616	A1	
7(ii)	$s=\frac{3}{2}\sin 2t-t$ $(+c)$	M1A1	**M1** for $\sin 2t$ and $\pm t$
	$t=\frac{\pi}{4}\to$ $s=1.5-\frac{\pi}{4}$ $(=0.715)$	A1	
7(iii)	$a=-6\sin 2t$	M1A1	**M1** for $-\sin 2t$
	$t=0.615$ $\to a=-5.66$ or -5.65 or $-2\sqrt{8}$	A1	condone substitution of degrees

Question	Answer	Marks	Guidance
8(i)	$\cos\alpha = \frac{1}{3}$ oe	M1	
	$\alpha = 70.5°$	A1	
8(ii)	speed $= \sqrt{3^2 - 1^2}$	M1	Pythagoras/trig ratio/cosine rule
	$\sqrt{8}$ or $2\sqrt{2}$ or $2.83\ \text{m s}^{-1}$	A1	
8(iii)	time $= \frac{50}{\textit{their}\sqrt{8}}$	M1	
	$\frac{25\sqrt{2}}{2}$ or 17.7s	A1	
8(iv)	*their* 8(iii) seen	B1	
	$BC = 10\sqrt{2}$ or 14.1 m or 14.2 m	B1	
9(i)	$\frac{\text{d}}{\text{d}x}(\ln x) = \frac{1}{x}$ and $\frac{\text{d}}{\text{d}x}x^3 = 3x^2$ or $\frac{\text{d}}{\text{d}x}x^{-3} = -3x^{-4}$	B1	seen
	Substitution of *their* derivatives into quotient rule	M1	
	$\frac{\text{d}}{\text{d}x}\left(\frac{\ln x}{x^3}\right) = \frac{x^3 \times \frac{1}{x} - 3x^2 \ln x}{x^6}$ oe	A1	correct completion
9(ii)	$\frac{\text{d}y}{\text{d}x} = 0 \rightarrow 1 - 3\ln x = 0$ $\ln x = \frac{1}{3}$	M1	equate given $\frac{\text{d}y}{\text{d}x}$ to zero and solve for $\ln x$ or x
	$x = \text{e}^{\frac{1}{3}}$	A1	seen
	$y = \frac{1}{3\text{e}}$	A1	seen
9(iii)	$\frac{\ln x}{x^3} = \int \frac{1 - 3\ln x}{x^4}\text{d}x$ oe	M1	use given statement in (i)
	$\int \frac{1}{x^4}\text{d}x = \frac{-1}{3x^3}$	B1	seen anywhere
	$\int \frac{\ln x}{x^4}\text{d}x = -\frac{1}{9x^3} - \frac{\ln x}{3x^3}$ (+C) oe	A2	**A1** for each term

Question	Answer	Marks	Guidance
10(a)	LHS $= \dfrac{\sin^2 x + (1+\cos x)^2}{\sin x(1+\cos x)}$	B1	correct addition of fractions
	$= \dfrac{1+2\cos x+1}{\sin x(1+\cos x)}$	B1	expansion and use of identity
	$= \dfrac{2(1+\cos x)}{\sin x(1+\cos x)} = 2\operatorname{cosec} x$	B1	factorisation and completion
10(b)(i)	$\operatorname{cosec}^2 y - 1 + \operatorname{cosec} y - 5 = 0$ $\operatorname{cosec}^2 y + \operatorname{cosec} y - 6 = 0$	M1	use of identity for $\cot^2 y$ to obtain quadratic in $\operatorname{cosec} y$
	$(\operatorname{cosec} y - 2)(\operatorname{cosec} y + 3) = 0$	M1	solve 3 term quadratic for $\operatorname{cosec} y$
	$\sin y = \dfrac{1}{2}$, $\sin y = -\dfrac{1}{3}$	M1	obtain values for $\sin y$
	$y = 30°, 150°, 199.5°, 340.5°$	A2	**A1** for 2 values
10(b)(ii)	$2z + \dfrac{\pi}{4} = \dfrac{5\pi}{6}$ or $\dfrac{7\pi}{6}$ (2.6…, 3.6…)	M2	**M1** equate to $\dfrac{5\pi}{6}$ **M1** equate to $\dfrac{7\pi}{6}$
	$z = \dfrac{7\pi}{24}$ or $\dfrac{11\pi}{24}$ (0.916, 1.44)	A2	**A1** for 1 value
11(i)	Other root $= 4$	B1	
	$f(x) = (x-3)(x-3)(x-4)$ $= x^3 - 10x^2 + 33x - 36$	M1	multiply out $(x-3)(x-3)(x \pm p)$
	$a = -10$ $b = 33$	A2	**A1** for each Can be implied by correct cubic
11(ii)	$x = 6,\ x = 6,\ x = 1$ $x = 2,\ x = 2,\ x = 9$ $x = 1,\ x = 1,\ x = 36$	B4	**B1** for each of first two sets **B2** for third set