



ADDITIONAL MATHEMATICS

4037/22

Paper 2

May/June 2018

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

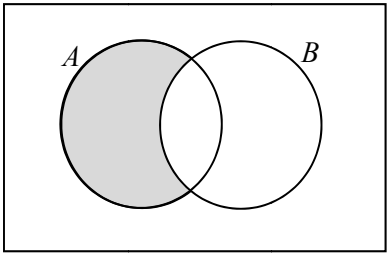
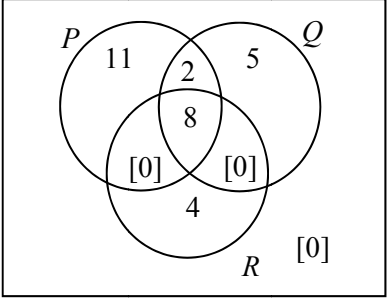
Types of mark

- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

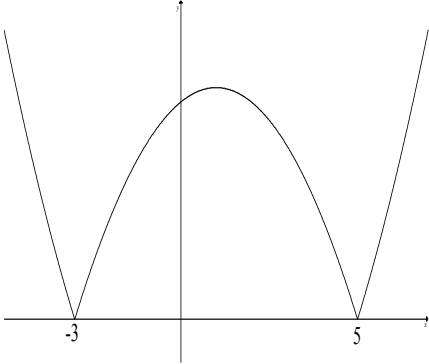
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1(i)	Uses $\cot \theta = \frac{\cos \theta}{\sin \theta}$ $\frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta}$ Uses $\cos^2 \theta + \sin^2 \theta = 1$ Completes to $\frac{1}{\sin \theta} = \operatorname{cosec} \theta$	B3	B1 for using $\cot \theta = \frac{\cos \theta}{\sin \theta}$ oe or $\tan \theta = \frac{\sin \theta}{\cos \theta}$ oe at some stage B1 for use of $\cos^2 \theta + \sin^2 \theta = 1$ oe B1 for common denominator of $\sin \theta$ oe either in a compound fraction or in two partial fractions or for writing $\frac{1 - \sin^2 \theta}{\sin \theta}$ as $\frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta}$ oe Maximum of 2 marks if not fully correct or does not complete to cosecθ
1(ii)	$\sin \theta = \frac{1}{4}$ 14.5° or 14.47[751...] rot to 4 or more figures isw	M1	
2(a)		B1	
2(b)		B3	B1 for 8 correctly placed and all the empty regions correct B1 for 11, 2, 5 correctly placed B1 for 4 correctly placed maximum of 2 marks if fully correct but other values such as 30, 21 and/or 15 present within the diagram
	<i>their</i> 12	B1	STRICT FT <i>their</i> Venn diagram

Question	Answer	Marks	Partial Marks
3	$p(-3) = 0$ or $p(2) = -15$ stated or implied	M1	
	$-54 + 9a + 72 + b = 0$ or better	A1	finds one correct equation; implies M1
	$16 + 4a - 48 + b = -15$ or better	A1	finds another correct equation; implies M1
	Solves a pair of simultaneous equations in a and b	M1	dep on first M1 condone one sign or arithmetic error in <i>their</i> solution; as far as finding one unknown
	$a = -7, b = 45$	A1	
	60 cao	A1	
4	Eliminates one of the unknowns	M1	
	Simplifies to a correct 3-term quadratic: $2x^2 + 4x - 16 [= 0]$ oe or $2y^2 - 6y - 36 [= 0]$ oe	A1	
	Factorises or solves $(x + 4)(x - 2) = 0$ oe or $(y + 3)(y - 6) = 0$ oe	M1	FT <i>their</i> 3-term quadratic in x or y ;
	$(2, 6)$ and $(-4, -3)$ oe	A2	Not from wrong working A1 for either $(2, 6)$ or $(-4, -3)$ or A1 for $x = 2$ and $x = -4$ or $y = 6$ and $y = -3$
5(a)	7P_4 or $7 \times 6 \times 5 \times 4$ oe	M1	
	840	A1	
5(b)(i)	20	B1	
5(b)(ii)	${}^5C_1 \times {}^4C_1 \times {}^2C_1$ or $5 \times 4 \times 2$ oe	M1	
	40	A1	
5(b)(iii)	${}^5C_3 + {}^4C_3$ oe	M1	
	14	A1	

Question	Answer	Marks	Partial Marks
6(i)	(Arc length =) 1.5×5 oe soi	M1	implied by 7.5
	($DE =$) $10\sin(0.75)$ oe soi	M1	implied by awrt 6.82
	34.3 or answer in range 34.31 to 34.32	A1	
6(ii)	(Area sector =) $\frac{1}{2} \times 5^2 \times 1.5$ oe	M1	implied by 18.75
	(Area triangle =) $\frac{1}{2} \times 5^2 \times \sin(1.5)$ oe	M1	implied by awrt 12.47
	31.2 or answer in range 31.21 to 31.22	A1	
7(i)	$ \mathbf{a} + \mathbf{c} = \sqrt{5^2 + 14^2}$	M1	
	$\sqrt{221}$	A1	mark final answer
7(ii)	$[(2 + m)\mathbf{i} + (3 - 5m)\mathbf{j}]$ therefore $2 + m = 0$	M1	for attempting to form $\mathbf{a} + m\mathbf{b}$ and equate the scalar of the \mathbf{i} component to 0
	$m = -2$ only	A1	implies M1
7(iii)	$[(2n - 1)\mathbf{i} + (3n + 5)\mathbf{j}] = 3\mathbf{i} + 11\mathbf{j}$ or $n(2\mathbf{i} + 3\mathbf{j}) = (3\mathbf{i} + 11\mathbf{j}) + (\mathbf{i} - 5\mathbf{j})$ oe leading to]	M1	
	$2n - 1 = 3$ or $3n + 5 = 11$ oe, soi $n = 2$ only	A1	implies M1

Question	Answer	Marks	Partial Marks
8(a)	$\begin{pmatrix} -2 & 6 \\ 1 & 12 \end{pmatrix}$	B2	B1 for a 2 by 2 matrix with 2 or 3 correct elements
	<i>their</i> $\left[\frac{1}{-30} \begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix} \right]$ oe isw	B2	<p>FT <i>their</i> non-singular BA</p> <p>B1 FT for either $\frac{1}{\text{their}(-30)} \begin{pmatrix} & \\ & \end{pmatrix}$ or</p> <p>$\dots \times \text{their} \begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$</p> <p>If <i>their</i> BA is singular, B0 then SC1 for</p> <p>$\dots \times \text{their} \begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$</p> <p>OR</p> <p>Alternative method $A^{-1}B^{-1}$:</p> <p>B2 for $A^{-1} = \frac{1}{-5} \begin{pmatrix} -3 & 1 \\ -1 & 2 \end{pmatrix}$ isw</p> <p style="text-align: right;">or $B^{-1} = \frac{1}{6} \begin{pmatrix} -5 & 2 \\ -3 & 0 \end{pmatrix}$ isw</p> <p>or B1 for a multiplier of $\frac{1}{-5}$ or for $\begin{pmatrix} -3 & 1 \\ -1 & 2 \end{pmatrix}$</p> <p style="text-align: right;">or for a multiplier of $\frac{1}{6}$ or for $\begin{pmatrix} -5 & 2 \\ -3 & 0 \end{pmatrix}$</p> <p>B2 FT for $A^{-1} B^{-1} = \text{their} \frac{1}{-30} \times \text{their} \begin{pmatrix} 12 & -6 \\ -1 & -2 \end{pmatrix}$</p> <p>or B1 FT for a 2 by 2 matrix with 2 or 3 correct elements</p> <p>Maximum of 3 marks if not fully correct</p>
8(b)(i)	2×3	B1	
8(b)(ii)	$\left(2 \quad -\frac{1}{2} \right)$ oe isw	B2	<p>B1 for each correct element; must be in a 1 by 2 matrix</p> <p>or M1 for a full method as far as finding values for the two elements</p>

Question	Answer	Marks	Partial Marks
9(i)	$\frac{d}{dx}(\sqrt{\sin x}) = \frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)$ oe	B2	B1 for $\frac{1}{2}(\sin x)^{-\frac{1}{2}} \times \dots$ or for $\frac{1}{2}(\sin x)^{-\frac{1}{2}}$ or for $\frac{1}{2}(\dots)^{-\frac{1}{2}} \times \cos x$ or for <i>their</i> $\frac{1}{2}(\sin x)^{\left(\frac{1}{2}\right)^{-1}} \times \cos x$
	<i>their</i> $(4x^3)\sqrt{\sin x}$ $+ x^4\left(\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)$ oe	M1	Applies correct form of product rule
	$4x^3\sqrt{\sin x} + x^4\left(\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)$ oe isw	A1	Not from wrong working
9(ii)	$\int(4x^3\sqrt{\sin x})dx$ $+ \int\left(x^4 \times \frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)\right)dx$ $= x^4\sqrt{\sin x}$ oe	M1	or $\int x dx + 2\int\left(\frac{x^4 \cos x}{2\sqrt{\sin x}} + 4x^3\sqrt{\sin x}\right)dx$ oe FT their (i)
	$\frac{x^2}{2} + 2x^4\sqrt{\sin x} [+c]$	A2	A1 for $\int x dx + 2x^4\sqrt{\sin x}$
10(a)(i)		B2	B1 for correct shape B1 for roots marked on the graph or seen nearby provided graph drawn and one root is negative and one is positive
10(a)(ii)	Any correct domain	B1	
10(b)(i)	$\frac{4}{3x-1}$	B1	mark final answer

Question	Answer	Marks	Partial Marks
10(b)(ii)	Correct method for finding inverse function e.g. swopping variables and changing subject or vice versa; or indicates $(hg)^{-1}(x) = g^{-1}h^{-1}(x)$ and finds $g^{-1}(x) = \frac{x+1}{3}$ and $h^{-1}(x) = \frac{4}{x}$	M1	FT only if <i>their</i> $hg(x)$ of the form $\frac{a}{bx+c}$ where a, b and c are integers
	$[(hg)^{-1}(x) =] \frac{1}{3} \left(\frac{4}{x} + 1 \right)$ oe isw or $[(hg)^{-1}(x) =] \frac{4+x}{3x}$ oe isw	A1	FT <i>their</i> $(hg)^{-1}(x) = \frac{a-cx}{bx}$ oe If M0 then SC1 for <i>their</i> $hg(x)$ of the form $y = \frac{a}{x} + b$ oe leading to <i>their</i> $(hg)^{-1}(x)$ of the form $y = \frac{a}{x-b}$ isw
10(c)	a cao	B1	
11(a)	$\frac{(2x-1)^4}{\frac{4}{3} \times 2} [+c]$ oe isw	B2	B1 for $k \times \frac{(2x-1)^{\left(\frac{1}{3}+1\right)}}{\left(\frac{1}{3}+1\right)}$ where $k \neq 0$
11(b)(i)	$k \cos 4x [+c]$ where $k < 0$ or $k = \frac{1}{4}$	M1	
	$-\frac{1}{4} \cos 4x [+c]$	A1	
11(b)(ii)	Sight of correct substitution of limits: $-\frac{1}{4} \cos \frac{4\pi}{4} - \left(-\frac{1}{4} \cos \frac{4\pi}{8} \right)$ oe	M1	FT <i>their</i> $k \cos 4x$ from (b)(i) dep on M1 awarded in (b)(i)
	$\frac{1}{4}$	A1	does not imply M1

Question	Answer	Marks	Partial Marks
11(c)	$\int e^{\frac{x}{3}} dx = ke^{\frac{x}{3}} [+c]$	M1	k any non-zero constant
	$k = 3$	A1	
	Sight of correct substitution of limits: $their ke^{\frac{\ln 8}{3}} - their ke^0$ oe	M1	dep on first M1
	Shows how to deal with the power of the first term e.g. $\frac{\ln 8}{3} = \ln 8^{\frac{1}{3}}$ or $\frac{\ln 8}{3} = \ln 2$ or $3(\sqrt[3]{8})$ seen	B1	
	$6 - 3 = 3$	A1	Not from wrong working
12(i)	$\tan \frac{\pi}{12} = \frac{r}{h}$ oe	M1	
	$r = h(2 - \sqrt{3})$ or $r = h \tan \frac{\pi}{12}$ oe	A1	
	$[V =] \frac{1}{3} \pi (2 - \sqrt{3})^2 h^2 \times h$ oe	M1	Correctly uses <i>their</i> expression for r in terms of h in formula for volume of a cone dependent on finding an expression connecting r and h
	$[V =] \frac{\pi(4 - 4\sqrt{3} + 3)h^3}{3}$ oe correctly leading to $[V =] \frac{\pi(7 - 4\sqrt{3})h^3}{3}$ AG	A1	
12(ii)	Correct derivative of V e.g. $\frac{3\pi(7 - 4\sqrt{3})h^2}{3}$ oe isw	B1	
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ soi	B1	
	$\frac{1}{their \left(\frac{dV}{dh} \right)_{h=5}} \times 30$	M1	if correct implies B1 B1 ; if incorrect, a correct FT statement implies the second B1
	5.32	A1	