MARK SCHEME for the October/November 2006 question paper

4024 MATHEMATICS

4024/02   Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

The grade thresholds for various grades are published in the report on the examination for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses.

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1 (a) (i) $5(x + 2)(x - 2)$ seen   B2   2   2

       After B0, allow B1 for partial factorisation, e.g. $5(x^2 - 4)$ or $(5x + 10)(x - 2)$

                            or $(x + 2)(x - 2)$ seen etc

    (ii) Final answer $\dfrac{x - 2}{2(x - 1)}$ oe including $\dfrac{x - 2}{2x - 2}$ asc   B2   2   2

       After B0, allow B1 for $\dfrac{\text{Their (a)(i)}}{10(x - 1)(x + 2)}$ soi or $\dfrac{5x - 10}{10x - 10}$ oe nww

                or quadratic factors of denominator including $(x - 1)(x + 2)$

   (b) $\dfrac{4(y + 5) - 3(y - 3)}{(y - 3)(y + 5)}$ oe   soi   M1

       If denominator in this form, inner brackets essential
       If not in this form, accept quadratic expression with y^2 and -15

       Final answer $\dfrac{y + 29}{(y - 3)(y + 5)}$ oe   A2   3   2

       After M1 A0, allow A1 for correct simplified numerator and denominator seen,
            not necessarily at the same stage

   (c)   **Final answer** $(g =) \dfrac{4 \pi^2 L}{T^2}$ oe   cao   B3

       Correct final answer involving an expression divided by fraction   SCB2

       or, in either order, Square their equation ft   M1

       and Clears fraction, $(gT^2 = 4\pi^2 L)$ ft   indep   M1   3   3   10

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2 (a) (i) **Final answer** $(9 , 6)$ or $x = 9, y = 6$ B1   1   1

     (ii) $\dfrac{3}{4}$ or $\dfrac{6}{8}$ or $(0).75$ B1   1   1

    (iii) $(\pm) 10$ B1   1   1

   (b) (i) **Final answer** $(-12 , 2)$ or $x = -12, y = 2$ Condone brackets missing   B2   2   2

        After B0, allow B1 for $\begin{pmatrix} -8 \\ 5 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \end{pmatrix}$ oe or $\begin{pmatrix} -12 \\ 2 \end{pmatrix}$

     (ii) Trapezium   indep   B1   1   1   6

3

(a) $91^2 = 53^2 + 64^2 \pm 2 \times 53 \times 64 \cos (P)$ oe soi   M1

$(\cos P =) \dfrac{53^2 + 64^2 - 91^2}{2 \times 53 \times 64}$ oe soi $(= \dfrac{-1376}{6784})$ $(= -0.2028)$ M1

$(P =) 101.65°$ to $101.75°$ A1 3 2

If only one or both of other angles alone found,

allow M1 for $53^2 = 64^2 + 91^2 \pm 2 \times 64 \times 91 \cos (Q)$ or $64^2 = 53^2 + 91^2 \pm 2 \times 53 \times 91 \cos (R)$

and A1 for $(Q =) 34.75°$ to $34.85°$ or $(R =) 43.45°$ to $43.55°$

Long methods : Allow M2 A1

(b) $\sin S = \dfrac{53 \sin 68}{74}$ $(= 0.66406)$ M1

$S = 41.55°$ to $41.65°$ A1

$P = 70.35$ to $70.45°$ or $112 -$ their S   ft   (dep on M1)   A1 3 2

Long methods : Allow M2 A1

(c) $\frac{1}{2} \times 53 \times 74 \sin$ (their P)   M1

1845 to 1855 (m²)   cao   A1 2 2 8

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4 (a) (XBY =) $150°$ B1 1 1

(b) XAD = XBY (= 150)   B1

XA = XB **and** AD (= BC) = BY   B1

Conclusion drawn and at least one reason shown SAS needed if too many facts dep B1 3 -

(c) AXD = BXY soi   B1

Convincingly shows DXY = $60°$ (= AXB) **AG**   dep B1 2 -

(d) States DX = XY   B1

Correctly concludes triangle DXY is equilateral   dep B1 2 - 8
or DY = DX and/or XY with a reason   B1
    triangle is equilateral   dep B1
(c)(d) together ΔDCY congruent to Δ ADX and/or Δ BXY   B1
        DY = DX and/or XY   dep B1
        ΔDCY is equilateral   dep B1
        Angle DXY = $60°$ dep B1
Numerical values used for other angles cannot gain credit

5 (a) (i) (\$) 825   B1 1 1

    (ii) (£) 625   B1 1 1

    (iii) $\dfrac{792 \times 1.44}{1.65}$ M1

         691.2 (euros)   A1 2 2

(b) (i) (\$) 16 200   B1 1 1

    (ii) (Their 16 200) × 1.08 × 1.08   oe soi   M1

       (\$) 18 895.68   [Accept 18 896, 18895.7, 18895 or 18900]   ft   A1 2 2

    (iii) Figures $\dfrac{\text{Their (b)(ii)} - 15\,000}{15\,000}$ (× 100) or $\dfrac{\text{Their } 1200 + 1296 + 1399.68}{15\,000}$ M1

       25.95 to 26.05 (%)   [Accept 26]   ft   A1 2 2

       or 125.95 to 126.05 (%)   SC B1

(c)   Use of $\dfrac{12 \text{ or } 100}{112}$ soi   M1

     (\$) 41 500   A1 2 2 11

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6 (a)   **Formula**   For numerical $\dfrac{p \pm \sqrt{q}}{r}$, (not ± p), seen or used,

       Allow B1 for $p = -12$ and $r = 14$ **and** B1 for $q = 452$ or $\sqrt{q} = 21.2..$ soi   B1 + B1

       Complete square   Allow B1 for $(a + 6/7)^2$ or $(a + 6/7)$ oe soi

           and B1 for 113/49 or square roots such as 1.5185..or 10.63../7

       Final answers   Allow B1 for each of 0.66 and - 2.38   nww   B2 4 2

          or allow B1 for **both** 0.661. and -2.375.. seen or 0.66 and -2.38 seen   (1)

(b)

    (i) $4x + 6y = 816$ **seen** (leading to $2x + 3y = 408$)   B1 1 -

    (ii) $3x + 5y = 654$ oe   seen   B1 1 1

    (iii) $x = 78$ **and** $y = 84$ B3 3 3 9

       After B0, allow B2 for one correct answer found with no wrong working

       After B0, allow M1 for correct method to eliminate one variable

       After $5x + 3y = 654$ in (ii), allow SC B2 for **both** $x = 82$ **and** $y = 81.3$ or better

7 (a) $2\pi \times 30^2$ $(= 1800\pi)$ $(=5655)$ soi   M1

        $2\pi \times 30 \times 70$ $(=4200\pi)$ $(=13194)$ soi   indep   M1

        Their 1800π + their $4200\pi + \pi \times 30^2$ (provided all areas)   indep   M1

        21 650 to 21 750 (cm²)   A1   4   3

        Note Use of $3\pi30^2$ may be taken as $2\pi30^2 + \pi30^2$, **unless** contradicted

               by the addition of extra $\pi30^2$, when M0, M1, M1,A0 possible

   (b) (i) $\frac{2}{3}\pi \times 30^3$ $(= 18000\pi)$ $(= 56549)$ M1

        Their $18000\pi + \pi \times 30^2 \times 70$ $(=81\,000\pi)$ $(=254469)$ (both volumes)   indep   M1

        254 to 255 (litres)   cao   A1   3   2

       (ii) $\underline{\text{Their (b)(i)}}$ $(= 84.8..)$ M1
            3

        1 minute 24.5seconds to 1 minute 25.5 seconds   cao   A1   2   2

      (iii) (Length =) Figures $\underline{\text{Their (b)(i)}}$ M1
                           $[\frac{1}{2}(0.4 + 0.6) \times 0.3]$

        Correct conversion of units   (using 1000)   indep   M1

        1.690 to 1.700 m or 169.0 to 170.0 cm [Unit essential in this case]   cao   A1   3   3   12

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8 (a) (i) 21 , 28   B1 1   1

(ii) $\frac{1}{2} \times 7 \times (7 + 1) = 28$ $(= T_7)$ or better seen   B1 1   -

(iii) 5050   B1 1   1

(iv) 25 250   or   5 × their (iii)   ft   B1 1   1

(v) Attempts to use T_{500} - their (iv)   (provided their (iv) < their T_{500})   M1

     100 000   A1 2   1

(b) (i) $S_6 = 56$ B1

     $S_7 = 84$ B1 2   2

     After B0 + B0, allow M1 for correct expansion of either or both expressions

(ii) $(7 \times (7 + 1) \times (7 + 2)) \div 6 = 84$ $(= S_7)$ or better seen   B1 1   -

(iii) 1540   seen   B1 1   1

(c) (i) $S_4 - S_3 = (1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 1) - (1 \times 3 + 2 \times 2 + 3 \times 1)$

         $= 4 + 3 + 2 + 1 (= T_4)$ seen   B1 1   -

     20 – 10 =10 is enough to score

(ii) $S_{n+1} - S_n = (n + 1) + n + (n - 1) + ... + 2 + 1 = T_{n + 1}$ justified   B1 1   -   12

     If algebraic methods used, mark strictly, expecting at least one step seen

9

(a) $\sqrt{\{104^2 - 100^2\}}$ or 28.56... oe seen [leading to 28.6 **AG**]   B1 1 -

(b) (i) 25°   B1 1 1

(ii) (FN =) 100 tan (their 25) (= 46.63..)   M1

(FB =) Their [46.63 - (28.56.. or 28.6)]   dep M1

18.00 to 18.10 (m)   [Expect at least 3 sig figs here]   A1 3 3

Alternative methods : M2 A1

(c) (i) CN = $\sqrt{\{100^2 + 60^2\}}$ or   BC = $\sqrt{\{104^2 + 60^2\}}$ M1

= 116.6.... soi   or   = 120.06....   soi   A1

tan BCN = $\dfrac{\text{Their } 28.6}{\text{Their CN}}$ or   sin BCN = $\dfrac{\text{Their } 28.6}{\text{Their BC}}$ M1

13.70° to 13.80°   cao   A1 4 3

Alternative methods : still M1 A1 M1 A1

(ii) BD = $\dfrac{(\text{Their } 28.6)}{\sin 10}$ (= 164 to 165)   M1

cos DBA = $\dfrac{104}{\text{Their BD}}$ (= 0.63.....)   dep M1

50.75° to 50.85°   cao   A1 3 2 **12**

or   DN = $\dfrac{(\text{Their } 28.6)}{\tan 10}$ (= 162.198)

and   DA = $\sqrt{\{\text{their } 162.198^2 - 100^2\}}$ (=127.7)   M1

tan DBA = $\dfrac{\text{their } 127.7}{104}$ dep M1

50.75° to 50.85°   A1

Alternative methods : M2 A1

10   Condone inaccuracies of up to 1 mm in plotting and drawing.

If plots are not visible, allow P marks if curve passes within 1 mm of correct plot.

Both P and dep C marks can be recovered following a grossly wrong plot if the

plot is ignored and the curve passes within 1 mm of the correct point.

Lined or plain paper used : no penalty, but extend tolerances to 2 mm.

Penalties , only to be applied to any P or C marks earned :

        Wrong scale(s) : - 1 once

        Interchanged axes : no penalty if labelled, - 1 otherwise

        Non-uniform scale(s) : - 2 after marking as generously as possible

(a)   8(.03)   B1 1   1

Ignore graph for $x < 1$ and for $x > 6$ throughout rest of question

(b)   All 7 points plotted   ft   (P1 for at least 5 of these   ft)   P2

Smooth curve, not grossly thick , through all plotted points, of which

                                  at least 5 are correct   C1 3   -

(c)

    1.35 to 1.45   B1

    3.55 to 3.70   B1 2   2

(d)   Drawing tangent at x = 4 and estimating $\dfrac{\text{change in y}}{\text{change in x}}$ M1

    1.20 to 1.40   A1 2   -

           Accept $\dfrac{\text{integer}}{\text{integer}}$ if in range for A1

(e) (i) Ruled straight line within 1 mm of both (1 , 3.5) and (5 , 5.5)   L2 2   -

    After L0, allow L1 for a good freehand line through these points,

    or a ruled line that would pass within 1 mm of the points if longer,

    or a ruled line that is long enough and passes within 2 mm of the points

  (ii)

    1.45 to 1.55 **and** 4.55 to 4.65   X1 1   1

  (iii) $2x^3 - 5x^2 - 30x + 50$ (= 0)   or any equivalent **equation**   E1 1   1 **12**

        Accept $a = -5$, $b = -30$ and $c = 50$