

# MATHEMATICS D

---

Paper 4024/11

Paper 11

## Key messages

To be successful in the examination, candidates need to have fully covered the syllabus, remember accurately all relevant formulae, use the appropriate methods for solving each problem (especially those involving multiple steps), accurately perform calculations and show clearly all relevant working for each question.

Writing answers in pencil first and then overwriting in ink is problematic and should be discouraged. Unwanted working or incorrect answers are best crossed out and replaced rather than overwritten – this was particularly noticeable on **Question 20**.

## General comments

The paper contained questions which were accessible to all candidates. In general candidates need to improve their skills in areas such as geometry, particularly topics involving circles and polygons, inequalities and non-linear equations represented graphically, and sets.

Questions involving single step answers were generally done well whilst those containing multiple steps or multiple concepts often proved a challenge to many candidates.

Presentation of the work was usually good, with most candidates showing sufficient clear working. The working in **Question 14(b)**, however, was frequently very difficult to follow.

Candidates should ensure careful reading of the question so that they are answering the question asked and that their answer is in the correct form. On a non-calculator paper, all necessary working should be shown, in ink, and pencil used only for diagrams and graphs. Erasing working completed in pencil also produces a great deal of detritus on the paper which results in lack of clarity in the work, particularly with negative signs and decimal points.

The answer line should contain only one answer and not a choice of answers. Care should be taken in transferring the answer found in the working onto the answer line.

There were a number of scripts which showed no attempt for the questions at the end of the paper. It was not clear whether this was due to lack of time or of knowledge of the topics involved.

## Comments on specific questions

### Question 1

- (a) This part was well answered with the common wrong answer being 4106.
- (b) A good proportion of answers were correct. Common wrong answers were 24, 23.7 or  $2.374 \times 10^5$ .

Answers: (a) 41 006; (b) 240 000.

### Question 2

- (a) Candidates who recognised that  $(-1)^2 = 1$  were successful. The common wrong answers were 8 (from  $10 - 2$ ) and 14 (from  $10 + 2^2$ ).

- (b) This part was answered well, with the answer given as either a decimal or a fraction. The common wrong answer was 0.8.

Answers: (a) 12; (b) 0.08.

### Question 3

- (a) There was a high proportion of correct answers. Occasionally candidates gave the answer as 0.03, not noticing that the answer had to be a fraction.
- (b) This part was answered well, with the common error being to carry out the subtraction before the division, giving an answer of 37. There were a number of arithmetic errors, often resulting in an answer of 72.

Answers: (a)  $\frac{3}{100}$ ; (b) 82.

### Question 4

- (a) The correct answer was fairly common but a number of candidates gave 4 or  $4^3$  as the answer.
- (b) There was some misunderstanding of the question, with answers of  $2 \times 2 \times 67$ , rather than 67, being given. Some candidates were able to factorise correctly in the working but were unsure what to write in the answer space and left it blank.

Answers: (a) 64; (b) 67.

### Question 5

The majority of candidates were able to score at least 1, usually for sight of  $c(2a - 3b)$ , but errors with signs resulted in the common wrong answer of  $(2a - 3b)(c - 2d)$ . There were a number of incorrect answers of  $(2a - 3b)(c + d)$ .

Answer:  $(2a - 3b)(c + 2d)$ .

### Question 6

- (a) Most candidates correctly converted the division into a multiplication. The common error was to then multiply the numerators but not the denominators, giving an answer of  $\frac{8}{3}$ . Occasionally the 3's were cancelled in the original multiplication resulting in an answer of  $\frac{1}{2}$ .
- (b) There was a common misunderstanding of the question as some candidates thought that the 21 mints were  $\frac{1}{4}$  of the sweets, rather than  $\frac{3}{4}$ , leading to the wrong answer of 84. Others found a quarter of 21 and added it to 21, giving a rounded answer of 26.
- (c) The majority of calculations found  $\frac{3}{8}$  and  $\frac{5}{8}$  of 360 and subtracted to give the correct answer, rather than simply finding  $\frac{2}{8}$  of 360. Wrong answers were often due to arithmetic errors.

Answers: (a)  $\frac{8}{9}$ ; (b) 28; (c) 90.

### Question 7

This simultaneous equation question was generally answered well. Errors generally occurred in the subtraction of the 2 equations, with  $y = 1$  being common. Those who used the substitution method were not as successful, with errors in multiplying out the bracket and in removing the denominator. A few attempted to use a determinant method, but were rarely successful.

Answer:  $x = 4, y = -1$ .

### Question 8

- (a) The common wrong answer was 8, ignoring the power of 1. The correct answer for  $n$  was 9, not  $3^9$ .
- (b) Those who found the fifth root of 32 first, and then cubed their answer, were generally successful. A few calculations of  $32 \times 3/5$  were seen.
- (c) This part was generally correct. The common wrong answer was  $\frac{1}{25}$  and the common unsimplified answer was  $\frac{1}{\sqrt[5]{25}}$ .

Answers: (a) 9; (b) 8; (c) 25.

### Question 9

This question, involving multi-topic concepts of circles and right-angled triangles, was generally not well understood. Many of those who recognised both the right angle at  $A$  and that  $OT = 6 + 4 = 10$ , and showed all relevant working, were able to score full marks. A number of candidates were given partial credit for the recognition of Pythagoras's theorem. Others tried to use 6 and 4 or used formulae involving  $\pi$ , trigonometry or areas. Many candidates either marked 6 cm on the diagram and could not proceed any further or did not attempt the question.

Answer: 8.

### Question 10

- (a) Expressing answers in set notation proves a difficult concept for many candidates. The common wrong answers were  $P \cap Q$ ,  $(P \cap Q) R'$  or  $(P \cap Q) \cup R'$ .
- (b) Most candidates did not recognise that, to find the lowest possible number of students in the group, those playing cricket also had to play football, i.e. 'cricket' taken as a subset of 'football'. The common wrong answers were 77 (from adding all the numbers given) and 9.

Answers: (a)  $P \cap Q \cap R'$ ; (b) 47.

### Question 11

- (a) Most candidates attempted the matrix multiplication but many did not add the relevant numbers together, giving either an unsimplified answer or a  $2 \times 3$  matrix. There were a number of errors in the calculations.
- (b) There were a number of correct answers, following a  $2 \times 1$  matrix in (a). The correct answer did not involve the number of hours or earnings/hour.

Answers: (a)  $\begin{pmatrix} 330 \\ 417 \end{pmatrix}$ ; (b) P shows the amount earned in Week 1 and Week 2.

### Question 12

- (a) The correct answer was common. Most calculations reached  $100 \times 9.3$  but were sometime spoilt by giving the final answer as 93 or 9300.

- (b) There was a good proportion of correct answers. A correct start to making  $b$  the subject of the formula, usually multiplying the formula by 2 or multiplying out the bracket, was given credit.

Answers: (a) 930; (b)  $\frac{2s - an}{n}$ .

### Question 13

Recognition that  $d = kv^2$ , with 8 and 5 correctly placed, usually resulted in the correct formula being found. The final answer, of the breaking distance, was sometimes incorrect due to using 40 rather than  $40^2$ , or converting  $\frac{5}{64}$  to an incorrect decimal. A few candidates reached the correct value for  $d$  but could not proceed further with the awkward fraction. Some reversed 5 and 8 in the formula connecting  $d$  and  $v$ .

Answer:  $d = \frac{5v^2}{64}$  and 125.

### Question 14

There was little recognition throughout this question as to practical costs of fruit, with many answers given in the thousands of dollars.

- (a) The combination of mixed units proved too difficult for weaker candidates with apples costing \$18 a common error. Another error was to forget to add the melon. It could have been easier to use  $\frac{3}{4} \times 2.4$  rather than  $0.750 \times 2.4$ .
- (b) Presentation of this part was often very difficult to follow. Candidates frequently misinterpreted the question, thinking that, for example, the cost price for 1 orange was \$25 so 100 would cost \$2500. There were also problems with using units consistently. There were a number of correct answers for the cost price (\$120), selling price (\$192) or the profit (\$72). These were not always used correctly to find the percentage profit with  $72 \div 192 \times 100$  and  $72 \div 100 \times 120$  seen. Some solutions incorrectly used the cost of one of each item.

Answers: (a) 3.65; (b) 60.

### Question 15

- (a) Many candidates were able to plot  $C$  accurately with a significant number of others measuring 7 cm correctly but giving angle  $CAB$  as  $50^\circ$ .
- (b) Finding the area of an obtuse angled triangle proved difficult for many candidates. The requirement here was to measure an appropriate perpendicular height and use the standard formula  $\frac{1}{2}bh$ . Candidates often tried to use an inappropriate method such as  $\frac{1}{2}ab \sin C$  or Hero(n)'s formula. The perpendicular to  $BC$  was sometimes incorrectly shown as the perpendicular to  $AB$  at  $A$ . Many candidates incorrectly found  $\frac{1}{2} \times 8 \times 7$ .

Answers: (a) Triangle  $ABC$  drawn with an acceptable  $C$ ; (b) 21 to 22 inclusive.

### Question 16

- (a) The correct line drawn was fairly common.
- (b) This part was less successful with a number of candidates not attempting it. Wrong lines included  $x + y = 2$  or lines with positive gradients. The line should cover the diagram given.

- (c) Answers were often difficult to follow as the shading was not always clear and the required region was not always labelled R. The lines  $x = -1$  and  $y = 2$  were not always drawn, although attempts were made to shade the correct side of the lines. Shading just the whole region required, rather than shading the regions for each individual inequality, would have made the answer clearer. Those who choose to shade out must make the region R clear. The bottom left triangle just outside the region was not always clearly excluded.

Answers: (a)  $x + y = 6$  drawn correctly; (b)  $2y + x = 4$  drawn correctly; (c) Correct region shaded.

#### Question 17

- (a) The volume of  $180 \text{ cm}^3$  correctly comes from the area of the trapezium multiplied by the scoop's width. As the answer is given, a clear accurate method is required. Many answers were either unclear or wrong. A considerable number of candidates did not attempt this part.
- (b) There were a significant number of candidates who gave no response. Of those who did make an attempt, some gained credit for correctly converting either 22.5 litres or  $180 \text{ cm}^3$  into the appropriate unit but most did not know how many  $\text{cm}^3$  there were in a litre. Few realised the question involved using cube roots – many just used a linear relationship between the ratio of volumes and ratio of lengths. Some simply substituted 22.5 into the formula used in (a).

Answers: (a) Valid method, with  $\frac{1}{2}(11 + 7) \times 4 \times 5$ , leading to 180; (b) 20.

#### Question 18

- (a) Most answers were fully correct with others giving 14 and then 42 or 47. The misunderstanding here was to multiply 4(th term) and 5(th term) by 3 and subtract 1 giving answers of 11 and 14.
- (b) Acceptable answers of 149 or 38 (for the term) were not common. Solving the equation  $4n - 1 = 150$  did not always lead to selecting the correct term with answers such as 147 and 39 (from rounding up 38.25).
- (c) (i) A common misunderstanding was to use the rule as a term to term rule, giving 26 as the third term or giving the sequence 1, 2, 5, 26. Others wrote  $n^3 + 2$ ,  $n^4 + 3$ ,  $n^5 + 4$ ,  $n^6 + 5$ .
- (ii) Candidates find writing an expression in terms of  $x$  difficult. Many answers were numerical, usually 24. Candidates did not see the connection to (i) despite the question using 'hence'.

Answers: (a) 14 41; (b) 149; (c)(i) 2 5 10 17, (ii)  $n^2 - 1$ .

#### Question 19

- (a) The answer was generally correct with common wrong answers of  $1.4 \times 10^9$ ,  $13.6 \times 10^8$  and  $1.36 \times 10^{-9}$ .
- (b) The common error was  $56 \times 10^8$ .
- (c) Common errors were  $7.9 \times 10^5$ ,  $793 \times 10^3$  or to subtract the numbers and the powers resulting in an answer of  $1 \times 10^2$ .

Answers: (a)  $1.36 \times 10^9$ ; (b)  $5.6 \times 10^9$ ; (c)  $7.93 \times 10^5$ .

#### Question 20

Recognition of equations appropriate for a particular graph shape was not well understood. The answer of C for (b) was correct the most often as the quadratic graph was more recognisable. The exponential curve was often confused with the cubic or hyperbolic curves. Answers were sometimes unclear as one letter was overwritten by another.

Answers: (a) F; (b) C; (c) B; (d) E.

### Question 21

- (a) (i) Alternate angles were frequently given as the answer although the phrase was sometimes written as 'alternative' or 'alternating' angles. Some answers gave 'alternate segment' or incorrectly mentioned corresponding angles.
- (ii) The main error was to use the isosceles triangle  $ABE$  with 2 angles of  $58^\circ$ . A number of candidates simply found  $(180 - 58)/2 = 61^\circ$ .
- (b) Those who correctly used the formula for the angle sum of a pentagon were given credit, with those recognising there were 2  $x$ 's (not 1) generally scoring full marks. Very often the sum of the interior angles of a pentagon was thought to be  $360^\circ$ . A few candidates worked with exterior angles but did not always remember to carry out the final subtraction from  $180^\circ$ .

Answers: (a)(i) ...alternate angles..., (ii)  $119^\circ$ ; (b) 120.

### Question 22

Using a statistical graph is still a concept students find difficult. Lines should be drawn on the graph to indicate how the median and other values have been found (in this case the number of students who spent between 50 and 80 minutes at the gym).

- (a) There were a number of correct answers with the most common error being to add the 2 values of 46 and 88, rather than subtract. The answer of 46 was also seen a number of times.
- (b) There were a good number of correct plots, dealing successfully with the interval change for the final two points, with an acceptable curve joining them. A number of candidates misread the vertical scale thus plotting the points with cumulative frequencies 6, 46 and 88 incorrectly. A few plotted at the mid-interval values rather than the upper class boundaries.
- (c) (i) Most gave the correct answer although there was some misreading of the horizontal scale. Others gave the answer 60 for the midpoint on the time axis or read up to the curve from 60 on the time axis.
- (ii) There were some good attempts but candidates should be encouraged to draw lines on their graph to show their method and also to show their calculation. There were a number of answers where the two values on the cumulative frequency axis were added together. Some candidates found the value mid-way between 30 and 80 on the time axis and read off the vertical scale at that point.

Answers: (a) 42; (b) Correct plots at 20, 40, 60, 90, 120 and cumulative frequency curve drawn; (c)(i) 62 to 64 inclusive, (ii) 41 to 46 inclusive.

### Question 23

- (a) A number of candidates realised that the first step was to find the numerical values for the vectors but not all of them knew how to draw the vectors on the diagram starting at  $A$ . A few developed a set of axes centred at  $A$  and plotted  $B$ ,  $C$  and  $D$  that way. Others drew vectors of the correct size but not starting at  $A$ . A number of candidates omitted this question.
- (b) A number of answers to (b) contained vectors such as  $\overrightarrow{PQ}$  rather than  $\mathbf{p}$  and  $\mathbf{q}$ .
- (i) There were some correct answers but also answers such as  $\mathbf{p} - \mathbf{q}$  and  $\mathbf{p} + \mathbf{q}$ .
- (ii) There were fewer correct answers here, partly due to candidates not simplifying their answer. Others did not attempt the question.
- (iii) Candidates who attempted this part often found (ii)  $+ 2\mathbf{p}$  rather than (ii)  $- 2\mathbf{p}$ , as they did not realise that the question needed  $\overrightarrow{RT}$  rather than  $\overrightarrow{TR}$ . Candidates who gave a first step such as  $\overrightarrow{OT} = \overrightarrow{OR} + \overrightarrow{RT}$  were generally more successful.

Answers: (a) Points  $B$ ,  $C$  and  $D$  marked correctly; (b)(i)  $\mathbf{q} - \mathbf{p}$ , (ii)  $\frac{2}{3}\mathbf{p} + \frac{1}{3}\mathbf{q}$ , (iii)  $\frac{1}{3}\mathbf{q} - \frac{4}{3}\mathbf{p}$ .

# MATHEMATICS D

Paper 4024/12

Paper 12

## Key messages

In order to do well in this paper, candidates need to

- have covered the whole syllabus
- remember necessary formulae and facts
- recognise, and carry out correctly, the appropriate mathematical procedures for a given situation
- perform calculations accurately
- show clearly all necessary working in the appropriate place.

## General comments

It was noticeable that there were many scripts where candidates had not attempted many questions and had made poor attempts at the others. Some candidates did not appear to bring geometrical instruments into the examination.

At times, candidates appeared not to read the questions carefully. They should be encouraged to pay careful attention to the wording, the numbers given, the units used and the units required in the answer.

Questions that proved particularly difficult were 2(b), 6(b), 9(b), 11(c), 14(b), 15(b), 16(b), 23(c) and 25(c)(ii).

It was noticeable that a significant number of candidates need to improve their ability to estimate, approximate and use appropriate degrees of accuracy; and also to understand the integer class of number. Some candidates were very competent at performing standard techniques, and yet seemed unable to recognise the appropriate mathematical procedure required for a given situation.

Candidates need to improve their skills in manipulating fractions and in basic arithmetic, particularly when negative numbers are involved. Work, that used correct methods, was spoilt by wrong calculations, such as  $\frac{2}{5} \times \frac{1}{4} = \frac{3}{20}$ ;  $\frac{1}{10} + \frac{3}{10} = \frac{4}{20}$ ;  $60 \times 60 = 1200$ ;  $6 - (1) \times (-1) = 5$ .

Candidates need to be aware of the advantage in simplifying fractions involving products by cancelling a number in the numerator with one in the denominator at the beginning rather than doing this, or even using long division, at a later stage. Thus  $\frac{90 \times 1000}{60 \times 60}$  is equal to  $\frac{90 \times 10}{6 \times 6}$  (at a first stage, by cancelling 10s), which in turn is  $\frac{10 \times 10}{2 \times 2}$  (by cancelling 3s), followed by  $5 \times 5$ , rather than by evaluating  $\frac{90000}{3600}$ .

One noticeable problem occurred when an expression is used, followed by some simplifying. Then an equals symbol is introduced so some sort of cross-multiplying can take place. For example

$$\begin{aligned}\frac{x}{360} \times \pi \times 100 \\ \frac{x}{36} \times \pi \times 10 \\ \frac{x}{36} = \pi \times 10 \\ x = 360\pi\end{aligned}$$

Presentation of the work was usually good. Candidates should bear in mind that it is to their advantage to make sure they provide sufficient working and that this working is set out neatly and legibly. Working makes it possible for marks, where they are available, to be awarded for correct methods and intermediate results.

A few candidates did not heed the instructions on the front page - to write in dark blue or black pen. Except for diagrams and graphs, candidates must not write in pencil. Nor must they overwrite pencil answers in ink, as this can be difficult to read. Some candidates wrote in pencil and even erased their workings, often producing rubber and paper debris that interfered with the clarity of other answers, particularly with a decimal point and a negative sign.

Candidates should be made aware that only their final answer to each question should be written in the answer space. Alternative offerings and working should not be written there. When an answer is to be changed, it is far better to delete and replace the original one rather than attempt to write over it.

Care must always be taken to ensure that answers obtained in the working are accurately transferred to the answer space.

### Comments on specific questions

#### Question 1

- (a) This part showed that some candidates needed to read the question carefully, and to note that the three drinks cost \$1.42 *each*, not \$1.42 for all three. Those who read the question correctly sometimes made an arithmetic mistake.
- (b) Attempts at this question showed that some candidates need to improve their ability to calculate the difference between two times. A common wrong answer was 3 hours 25 minutes from 10 to 1 = 3 hours,  $45 - 20 = 25$  minutes. Some calculated  $10.45 - 1.20$ . Others calculated  $13.30 - 10.45 = 2.75 = 2$  hours 75 minutes = 3 hours 15 minutes.

Answer: (a) 5.11; (b) 2 hours and 35 minutes.

#### Question 2

- (a) Candidates who realised that there are 3 portions of 500 g in 1.5 kg usually went on to find the correct answer, from  $13 \times 3 + 20$ . Some obtained 99, from  $3 \times (13 + 20)$ ; others omitted to add 20 to their 39.
- (b) A minority of candidates was able to provide the correct formula. Some gave  $\frac{13M}{500}$  only, or  $M + 20$ , or  $\frac{33M}{500}$ , or  $T \times M$ .

Answers: (a) 59; (b)  $T = \frac{13M}{500} + 20$ .

#### Question 3

A few candidates did not seem to understand the difference between a median and a mean.

- (a) Some did not realise that to obtain the median, it is necessary to put the numbers in numerical order. Others did not seem to realise that the median is halfway between the centre pair of values, -1 and 0.
- (b) Most answers indicated the use of a correct method, but errors in arithmetic were quite common.

Occasionally, the value  $\frac{1}{10}$  was obtained in the working and 10 written in the answer space.

Answers: (a) -0.5; (b) 0.1.



**Question 4**

- (a) The majority of candidates knew to substitute  $-\frac{1}{2}$  for  $x$ .

Errors were sometimes made with signs by writing  $1 - 6 = 5$ , or by writing

$$2\left(\frac{1}{2} - 3\right) = 2 - 6, \text{ or } 1 - 3, \text{ or } 0 - 3.$$

- (b) Most candidates seemed to know how to deal with this type of question. A common wrong answer was  $\frac{x+3}{2}$ . A few obtained  $\frac{x+6}{2}$  and then spoilt their answer by cancelling 6 and 2 to get  $x + 3$ . Some looked to find  $f^{-1}(5)$  or similar, resulting in a numerical value only.

Occasionally, an answer was given that was not expressed in terms of  $x$ .

Answer: (a)  $-5$ ; (b)  $\frac{x+6}{2}$ .

**Question 5**

- (a) Answers like 12, 1200.000, 1200.0, 12345.67 show that some candidates need to gain a better understanding of significant figures, and their difference from decimal places.

- (b) Some candidates noticed, without any calculation, that  $28/\pi$  is approximately equal to 9, with a square root of 3. Others arrived at the approximate value of 9 from the calculation  $\frac{28}{\frac{22}{7}} = \frac{98}{11}$ . Some answers were left as an unresolved square root with a complicated fraction inside.

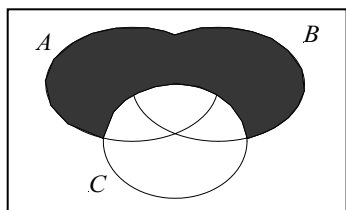
Answer: (a) 1200; (b) 3.

**Question 6**

- (a) Answers were very varied, and often included shading  $C'$  also. Some shaded  $C \cap A \cap B$ .

- (b) Many gave a set of values rather than  $n(Q)$ . A minority of candidates worked out that  $Q = \{0, 1, 4\}$ , and went on to give the correct answer.

Answer: (a)  (b) 3.



**Question 7**

Most candidates attempted this question well. Some candidates did not realise that there are 1000 metres in one kilometre and  $60 \times 60$  seconds in one hour. As mentioned above, some made hard work of the simplification of their expression. A few successfully quoted and used the multiplying factor  $5/18$ . Some multiplied by  $18/5$  to get 324. It was not uncommon to see calculations such as  $\frac{9000}{3600} = 40$ ;  $60 \times 60 = 1200$

Answer: 25.

### Question 8

Some candidates seemed to assume that the given ratio was for the *radii* and not for the *areas*.

- (a) Some candidates knew that the square roots of the ratio of the areas was needed. Many gave the wrong answers 1 : 4 or 1 : 16.
- (b) Some candidates knew to give the cubes of their part (a) answer. Others gave 1 : 4 in both parts (a) and (b).

Answers: (a) 1 : 2; (b) 1 : 8.

### Question 9

Responses indicated that many candidates did not have a strong understanding of bounds.

- (a) Some candidates obtained the correct answer. Some did not realise that the answer is obtained from  $54.3 - 0.05$ . Common wrong answers were 53.8, from  $54.3 - 0.5$ ; 54.2, from  $54.3 - 0.1$ .
- (b) Some candidates seemed not to realise that the first line of the whole question, in which the time was given, applied to part (b) as well as to part (a). A few candidates gave, correctly, the distance travelled as the upper bound,  $d + 0.5$ . The time taken was either given as 54.35, or correctly, as the answer to part (a).

Answers: (a) 54.25; (b)  $\frac{d + 0.5}{54.25}$ .

### Question 10

Most candidates dealt competently with this question. Some made errors in multiplying 9 by 8 or in dividing 72 by 6. A few assumed that  $y$  was inversely proportional to the *square* of  $x$ ; or else was *directly* proportional to  $x$ .

Answer: 12.

### Question 11

- (a) This part was usually answered correctly.
- (b) Most candidates gave the correct answer.
- (c) Whereas some candidates realised that a square function was involved, few gave the correct answer.

Answers: (a) 1; (b) 41 40 81; (c)  $(2n + 1)^2$ .

### Question 12

This was generally answered well by those who know what is meant by standard form.

- (a) The common wrong answers were  $5.67 \times 10^4$ ;  $567 \times 10^{-6}$ ;  $5.7 \times 10^{-4}$ .
- (b) The common wrong answers were  $\frac{3}{5} \times 10^{-11}$ ;  $0.6 \times 10^{-11}$ ;  $6 \times 10^{12}$ ; or to give 1.66 or 1.5 instead of 6.

Answers: (a)  $5.67 \times 10^{-4}$ ; (b)  $6 \times 10^{-12}$ .

### Question 13

(a) Many achieved the correct answer of 140, although some then went on to give their answer as 40 or 14. Others wrote  $\frac{7}{5} \times 100 = \frac{700}{5}$ , but could not evaluate this to get 140. Common wrong methods included  $\frac{5}{7} \times 100$  or  $\frac{7}{12} \times 100$ .

(b) The most popular method was to use an equivalent to  $\frac{x}{3} = \frac{7}{5}$  and obtain 4.2. Many seemed to think that this gave the length of  $ED$ , instead of  $AD$ . Some found  $ED$  directly by using  $\frac{x+3}{3} = \frac{7}{5}$ . A common error was to evaluate  $\frac{21}{5}$  as 4.5.

Answers: (a) 140; (b) 1.2.

### Question 14

Most candidates did not have a strong understanding of how a sector of a circle can be folded to produce a hollow cone. A large proportion of candidates offered nothing worthy of credit, or else omitted this question.

(a) The common wrong answer was 6 cm.

(b) Few candidates realised that they needed to equate the arc length of the sector ( $\frac{x}{360} \times 2\pi r$ , with  $r = 10$ ) to the circumference of the base of the cone ( $2\pi \times 6$ ), and that there is no need to use an approximation for  $\pi$ , as the  $\pi$  terms can be “cancelled out”.

Alternatively, the area of the sector could be equated to the curved surface area of the cone.

Some candidates used an incorrect formula for a length or for an area. Others calculated the *volume* of the cone.

Answers: (a) 10; (b) 216.

### Question 15

Candidates need to be aware that when a question deals in multiples of  $\pi$ , it is much better not to use an approximate value for  $\pi$ .

(a) Whereas many tried to use the given formula, relatively few could simplify their expression to obtain the correct value for  $k$ . Common wrong answers were 36, where candidates found the volume of only one sphere; 240, or where the cube of 3 was evaluated as 9.

(b) The majority of candidates seemed to have difficulty in visualising the situation.

The easiest method was to divide the volume of the 20 spheres by the area of cross-section of the cylinder.

Many candidates opted to give the initial depth of water a value, and then calculate the depth of water remaining after the spheres were taken out. Some of these overlooked the necessity of subtracting this value from the original depth in order to find the *change in depth* requested in the question. It was quite common to see something like  $\pi \cdot 6^2 \cdot 60 - 720\pi = 1440\pi$ , with  $1440\pi$  written as the answer.

Answers: (a) 720; (b) 20.

### Question 16

- (a),(b) Some candidates made a reasonably good attempt at these parts, though errors in signs occurred quite frequently. Methods included use of a matrix for the reflection - though often with a post-multiplication; use of vectors - though with confused directions; use of a drawing on the diagram - though with errors in converting coordinates to a vector.
- (c) Some candidates knew how to find the magnitude of a vector, though a few gave an answer that included a negative value. The common error was  $(-3)^2 = -9$ .

Answers: (a)  $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$ ; (b)  $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ ; (c) 5.

### Question 17

Candidates who knew the laws of indices answered this question well. Many showed that they need to improve their knowledge of these laws.

- (a) The usual errors were to give  $p^6$  for  $p^2 \times p^3$ ;  $p$  for  $p^0$ ;  $\frac{1}{3}$  for  $-3$ .
- (b) The usual errors were to give the constant 9 instead of 3; to give the power of  $x$  as 3 or 6.

Answers: (a)  $p^5 - 3$ ; (b)  $3x^2$ .

### Question 18

- (a) There were many correct solutions to this part. Some gave only a partial factorisation, such as  $2a(2 - 8a)$ . The usual errors involved treating the expression as the “difference in two squares”, or as a trinomial with factors such as  $(1 - 2a)(4a + 8a)$ .
- (b) Many realised that this part involved the “difference of two squares” and gave the correct factors. Common wrong answers were  $(3b - c)^2$ ;  $(9b + c)(9b - c)$ ; putting 9 or 3 outside the two brackets.
- (c) Most candidates showed that they could answer this part correctly, arranging the expression into two pairs so that each pair had a common factor. A partially factorised form,  $x(x - y) - 5(y - x)$ , for example, was sometimes presented as the answer. A frequent wrong answer was  $(x - 5)(x + y)$ .

Answers: (a)  $4a(1 - 4a)$ ; (b)  $(3b - c)(3b + c)$ ; (c)  $(x + 5)(x - y)$ .

### Question 19

- (a) Some candidates did not attempt this part, showing that they need to acquire a stronger understanding of rotational symmetry. Others gave answers of 2, 6 or 1.
- (b) This part was often omitted, or attempted with little understanding. Candidates who made a reasonable attempt usually obtained the 90 and the 150s. The 135s were less frequently obtained.

Answers: (a) 4; (b)  $90^\circ$  two  $150^\circ$  two  $135^\circ$

### Question 20

Many candidates showed that they were familiar with the angle properties of parallel lines and polygons drawn in a circle.

Many candidates obtained answers that they should have realised were clearly the wrong size just by looking at the diagram. For example, giving an obtuse or a reflex angle for something that is clearly acute.

Some otherwise correct attempts were spoiled by arithmetic errors.

- (a) The common error was to divide 136 by 2 incorrectly. Some gave  $224^\circ$ , from  $360^\circ - 136^\circ$ ;  $112^\circ$ , from  $(360^\circ - 136^\circ)/2$ ;  $44^\circ$ , from  $180^\circ - 136^\circ$ ;  $272^\circ$ , from  $2 \times 136^\circ$ .
- (b) Some candidates did not realise that, because  $CO$  is parallel to  $DA$ ,  $y = 180 - 136$ .
- (c) Many candidates realised that  $z^\circ$  and  $x^\circ$  are opposite angles in a cyclic quadrilateral and so sum to  $180^\circ$ . Frequent wrong answers were  $44^\circ$  and  $136^\circ$ .
- (d) Some candidates noted that in quadrilateral  $TAOC$ , the tangents and radii formed angles of  $90^\circ$ , or that  $TO$  bisected the quadrilateral. Others gave the answer  $68^\circ$ .

Answers: (a) 68; (b) 44; (c) 112; (d) 44.

### Question 21

The fact that this question was sometimes omitted, or answers given were whole numbers without any working, suggests that some candidates need to gain a better understanding of probability.

Some candidates did not seem to read the third line of the question, that if the first ball was red it was *not* put back into the bag.

Quite often candidates started with a correct method but manipulated their fractions incorrectly.

Occasionally an answer was not expressed in its simplest form, as directed in the question.

- (a) Most candidates drew two correctly labelled branches on the diagram. The usual errors were to give two probabilities of  $\frac{2}{4}$ , or  $\frac{1}{4}$  for blue and  $\frac{3}{4}$  for red.
- (b)(i) There were many correct answers to this part. Some did write down  $\frac{2}{5} \times \frac{1}{4}$ , but either evaluated this as  $\frac{3}{20}$  or did not simplify  $\frac{2}{20}$ .
- (ii) Many candidates worked correctly from their part (a) answer. Some added together the two probabilities given on the Second ball blue branches. Others used  $(\frac{2}{5} + \frac{1}{4}) \times (\frac{3}{5} + \frac{2}{4})$  or obtained the fractions  $\frac{1}{10}$  and  $\frac{6}{25}$  but failed to add them correctly.

Answer: (a) Correct labels and  $\frac{2}{5}$  with blue;  $\frac{3}{5}$  with red (b)(i)  $\frac{1}{10}$ ; (b)(ii)  $\frac{17}{50}$ .

### Question 22

Attempts at this question showed that some candidates need to get a better understanding of the properties of a speed-time graph, and to appreciate that the “D-S-T triangle” is not a valid method when there is an acceleration or retardation.

- (a) Often attempted correctly. Occasionally  $\frac{12}{10}$  became 0.12. Some candidates calculated the acceleration. Others used  $\frac{12}{60}$ .
- (b) Usually answered correctly by those who used  $12 \times \frac{9}{30}$ , particularly when this expression was cancelled down to  $\frac{18}{5}$  rather than trying to evaluate  $\frac{108}{30}$ .
- (c) Many candidates showed that they knew the distance travelled is equal to the area under the graph. Most found this area as two or three separate regions, or as one trapezium. A few made arithmetic errors in their calculations. Weaker responses offered 720, from  $12 \times 60$ .

Answers: (a) 1.2; (b) 3.6; (c) 480.

### Question 23

Candidates who could correctly identify how the three equations related to the points *A*, *B* and *C* usually made a reasonably good attempt at this question.

- (a) The key to this part was to solve the equations  $x = 8$  and  $2y = 12 + x$ .
- (b) Most candidates identified the two correct lines. Errors were frequently made with the inequality symbols.
- (c) Some candidates could find the coordinates of the required point, but many could not. Candidates were expected to work from the point *B* to find the point that satisfied all three inequalities. Some candidates needed a better understanding of the word *integer*.

Answers: (a) (8, 10); (b)  $x > 8$   $2y > 12 + x$ ; (c) (9, 11)f.

### Question 24

This question was quite often omitted, or revealed that many candidates needed a firmer understanding of loci and the difference between perpendicular bisectors of sides and bisectors of angles. Those who understood loci usually did quite well, drawing the loci to an acceptable degree of accuracy, and shading the appropriate region.

- (a) Often correct. A few candidates seemed to be trying to find the bearing of *A* from *B*.
- (b) Of those who attempted this part, most drew an appropriate part of a circle. The perpendicular bisector of the line *AB* was less successful. Some seemed to work from triangle *ABC*, or from parts of circles centred on *A* or *B*.

Those who constructed good loci usually shaded the correct region.

Answers: (a)  $137^\circ$  to  $139^\circ$ ; (b)(i) Perp. bisector of *AB*; (b)(ii) Circle, centre *C*, radius 4 cm; (b)(iii) Correct region (bottom part) shaded.

### Question 25

- (a) Usually attempted correctly, though careless errors in subtraction or in dividing by 2 occurred quite frequently. A common misconception was to give the  $x$  coordinate as 3.5 or the  $y$  coordinate as 3 - being the difference between the coordinates of  $P$  and  $M$ .
- (b) Usually attempted correctly, though careless errors in subtraction or in manipulating negative signs occurred quite frequently. Common wrong answers were 6;  $\frac{6}{7}$ ;  $-\frac{7}{6}$
- (c) Those candidates who drew a sketch to help visualise the situation were more successful than those who did not, especially in part (c)(ii)
- (c) (i) There was a wide variety of answers. Some tried to use the midpoint formula, others used the fact that the step from  $P$  to  $Q$  is equal to the step from  $Q$  to  $R$ .
- (ii) There were few correct answers to this part. Few candidates took the hint to "Write down", which suggests that the result should be obtainable by a minimum of calculation. Most attempts were rather complicated, involving, for example, the distance between two points formula (not always quoted correctly). Some wrote down a quotient of one pair of coordinates divided by another pair.

Answers: (a)  $(-\frac{1}{2}, 1)$ ; (b)  $-\frac{6}{7}$ ; (c)(i)  $(10, -8)$ ; (c)(ii)  $\frac{1}{3}$ .

### Question 26

- (a) Most candidates realised that  $k$  was related to the determinant of the matrix  $\mathbf{A}$ .  
The calculation  $6 - (1) \times (-1)$  was sometimes evaluated as 5, which was offered as the value of  $k$ . The usual wrong answer was 7.
- (b) Most candidates attempted to find  $\mathbf{X}$  correctly, but errors in subtraction and with negative signs were very common.
- (c) Few candidates seemed to know that if  $\mathbf{YA} = (6 \ 2)$  then, by post-multiplying by the inverse of  $\mathbf{A}$ ,  $\mathbf{Y} = (6 \ 2) \times \mathbf{A}^{-1}$ .

Some candidates realised that the matrix  $\mathbf{Y}$  must have one row and two columns. They wrote  $\mathbf{Y}$  as  $(a \ b)$ , multiplied out  $(a \ b) \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ , equated the answer to  $(6 \ 2)$  to give a pair of simultaneous equations, which they then solved.

Many candidates attempted either to divide  $(6 \ 2)$  by the matrix  $\mathbf{A}$ , or to replace a row matrix by a column matrix, and perform an unconventional method of matrix multiplication or division.

Answers: (a)  $\frac{1}{7}$ ; (b)  $\begin{pmatrix} -1 & -4 \\ 2 & 0 \end{pmatrix}$ ; (c)  $(2, 0)$ .

# MATHEMATICS

---

Paper 4024/21

Paper 21

## Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and remember necessary formulae. All working must be clearly shown, and, in numerical work, a suitable level of accuracy must be used in any given calculation, and maintained throughout a series of linked calculations.

## General comments

Scripts covering the whole mark range were seen. Many were well presented, using the working and answer spaces effectively. Clarity and neatness in presentation remain a problem for some candidates. All working must be done in blue or black ink; it should not be written in pencil and erased or over-written.

The paper enabled candidates at all levels to show what they knew.

Most candidates attempted **Question 7** in **Section B**.

**Question 9** was the least popular optional question. Many attempts at this question were abandoned on reaching part **(f)**.

Very few candidates understood how to apply the concept of upper bound in the context of **Question 4(b)**.

## Comments on specific questions

### **Section A**

#### **Question 1**

- (a) (i) The correct process was usually seen. A common error was the evaluation of  $\frac{18 \times 300}{100}$ .
- (ii) Again candidates usually understood the process required. The demand asking for the answer as a fraction in its lowest terms was not always fulfilled. Fractions such as  $\frac{0.1}{50}$  or  $\frac{0.6}{500}$  were not accepted.
- (iii) There was no problem in understanding how to evaluate 15% of a quantity. The mistake commonly seen was to find 15% of 300g rather than 15% of 18g.
- (b) The context here was a practical one, to find the number of 300g tins necessary to have 2500g of soup. Common errors were either the failure to appreciate that a whole number was required, or to round down to the nearest whole number, ending up with less than the amount of soup required.
- (c) (i) There were correct answers seen from those candidates who realised that this was a reverse percentage. Candidates who did not, often evaluated  $0.8 \times 4.2$ . They did not offer any attempt at an algebraic solution, such as  $x + \frac{20x}{100} = 4.2$ , that could have been constructed from the description of the question.



- (ii) A variety of approaches were possible, such as comparing cents per gram for the two tins, or comparing the cost of a given number of grams using each tin. A wide range of solutions were seen. A common error was to take the size of the promotional tin to be 300g. There were some solutions that showed all the necessary arithmetic but did not offer any written conclusion. In some cases, decisions were made from working that was either incomplete or not valid. There were a number of solutions that were presented haphazardly and hence were difficult to follow.

Answers: (a)(i) 6; (ii)  $\frac{1}{500}$ ; (iii) 2.7; (b) 9; (c)(i) 3.5; (ii) The special promotion tin with for example both 0.222 cents per gram and 0.224 cents per gram stated.

### Question 2

- (a) Candidates who began with  $21\ 50 + 6$  seemed to make fewer errors than those who began with  $21\ 50 + 11\ 15$ . In this case, the problem was in arriving at, and dealing accurately with,  $33\ 05$ .
- (b) Whatever order was adopted, the solution depended on subtracting a time involving 45 minutes from a time involving 40 minutes. This was the point at which an error most commonly occurred.
- (c) (i) Candidates understood how to use the graph given in the question.  
(ii) The demand here was usually well understood, with many candidates using correctly the information gained in part (i).
- (d) There were a good number of correct solutions seen. Some candidates used an incorrect number of multiples of 24, such as 29. There were some candidates who thought that ideas of proportionality were appropriate, such as  $\frac{29}{23}$  of 683.

Answers: (a) 15 05; (b) 11 hours 55 minutes; (c)(i) 290; (ii) 45; (d) 827.

### Question 3

- (a) (i) A lot of accurately drawn parabolas were seen. Care was needed in plotting  $y$  values since on the scale given, there were two small squares representing 0.1. There were some errors in finding the  $y$  value corresponding to  $x = -4$ . Common errors were  $-16$ , 0 or 10.  
(ii) This part was well understood. This time, care was needed reading the horizontal scale, since one small square represented 0.2.  
(iii) A tangent drawn at the correct point was usually accompanied by a correct calculation of the gradient. In some cases, the gradient turned out to be negative, and some candidates drew tangents at  $x = -3$ . Some attempts at tangents at  $(3, -5)$  clearly went through  $(4, 0)$ . These were not accepted since they were clearly chords of the parabola rather than tangents.
- (b) Usually well understood, with candidates earning good marks. There were some correct answers given that came from incorrect work. These were not accepted.
- (c) Attempted successfully by a minority of candidates. There were some candidates who compared the relevant quadratic expressions to arrive at the line  $y = -3x + 1$ . There were a few resourceful solutions that thought the ideas through in a different way. The points of intersection of  $L$  and the curve  $y = x^2 - 2x - 8$  were read off from the graph using the  $x$  values obtained in the previous part. The equation of  $L$  was then constructed using these two points. This part was frequently omitted.

Answers: (a)(i) 43.20; (ii)  $-2.3\ 4.3$ ; (iii) 4; (b) 2.54  $-3.54$ ; (c)  $y = -3x + 1$ .

#### Question 4

- (a) Some candidates were able to calculate both  $p$  and  $q$ . Those achieving one correct value usually got the 12, with 32 instead of 16 for the other value. This looks as though the width of the second column was found by counting squares rather than using the given scale. This part of the question was often omitted.
- (b)(i) The process required here was generally well known. A common error in the numerator was to multiply all the frequencies by 5. A common error in the denominator was division by 5.
- (ii) The correct answer was very rarely seen. The use of 17.5, 27.5 etc. was also seen very rarely. Sometimes 15.5, 27.5 etc. were used. A common misunderstanding was to add 2.5 to the numerator found in the previous part.

Answers: (a)  $p = 12$   $q = 16$ ; (b)(i) 29.5; (ii) 2070.

#### Question 5

- (a) The use of the cosine rule was generally appreciated, and it was often evaluated correctly. The final mark was not awarded unless 19.46 was shown.
- (b) Many candidates used the sine rule as expected. It was generally well known. There were a number of candidates who either omitted "sin" from their initial statement, or who dropped it before the end. A number of candidates opted to use the cosine rule again in this part.
- (c) Many candidates who perhaps would have been able to find the area of a triangle using the area sine formula had this been set out in a given triangle, were not able to relate this idea to the problem here. As a result, there were relatively few complete solutions seen. Common misunderstandings included setting the area of the rhombus as the area of triangle  $BCE$ , assuming that  $ACD$  was a straight line, and using 19.5 as the length of  $AE$ .

Answers: (a) 19.46 seen; (b) 37.5; (c) 248.

#### Question 6

- (a) This part was usually correct.
- (b) Again, this was often correct. As usual when transpositions of equations are involved, care is needed when moving terms from one side to the other.
- (c) Common misunderstandings here included giving  $f(3g)$  the value  $3g$ , or even treating  $f$  as an additional variable.

Answers: (a)  $-1$ ; (b)  $\frac{x+7}{2}$ ; (c) 2.2.

#### Section B

#### Question 7

- (a)(i) Candidates usually solved this equation correctly.
- (ii) Many candidates negotiated the algebra required here correctly. As usual, care was needed when removing brackets with minus signs involved.
- (b) Again, many candidates were able to bring this part to a successful conclusion. The numerator was generally factorised fully. The factor 5 was not always removed from the denominator, so that the difference of two squares was not always seen by candidates.
- (c)(i) A valid expression was often seen. In some cases, the necessary brackets were omitted. A common error was the answer  $20p$ .

(ii) There was a mixed response to this question. Candidates obtaining full marks here were careful to show how all 35 slabs were involved, and accurately converted units were appropriate. There were a number of unsuccessful attempts that failed on one or both of these counts. Some candidates attempted to solve the given equation at this stage.

(iii)(a) This was generally well done. Despite the demand in the question, the use of the formula was seen.

(b) Quite well done. Some candidates gave the width as their answer.

Answers: (a)(i) 0.75 (ii)  $-4$  (b)  $\frac{3w}{w+2}$  (c)(i)  $p(p+20)$  (ii) The given equation correctly derived.  
(iii)(a)  $p = 50$   $q = -70$  (b) 70

### Question 8

(a) (i) Candidates generally understood what was being asked. There were some curious answers, including lengths.

(ii) Candidates understood that the perpendicular bisector of  $AB$  was required. It was usually constructed accurately.

(iii)(a) There were a good number of candidates who drew a correct arc here. Quite a number were confused as to the region to shade. A common error was to see a line parallel to the perpendicular bisector instead of an arc.

(b) This was clear to those candidates who had a correct drawing at this point.

(iv) Again, what was required here was generally clear to those candidates who had constructed accurate drawings. A common error was to show the path of  $D$  starting from  $A$ . Many candidates made no response to this part of the question.

(b) (i) Most candidates were able to apply Pythagoras' theorem correctly in the right angled triangle  $PSR$ . The expected answer was 9.43, not 9.4.

(ii) This proved to be difficult for most candidates. A common misunderstanding was to use the midpoint of  $PQ$  as the shortest distance from  $S$ .

Answers: (a)(i) 112 to 116; (ii) Perpendicular bisector of  $AB$ ; (iii)(a) Correct region shaded; (b) 2.8 to 3.2; (iv) Yes as path of  $D$  passes through the shaded region; (b)(i) 9.43; (ii) 6.39.

### Question 9

(a) This was often correct. A common error was the answer 1.

(b) Again, many correct answers were seen. Common errors were to reflect triangle  $A$  in either the  $x$  or the  $y$  axis.

(c) Most candidates realised that  $-2.5$  was involved. Here, it was essential to state  $x = -2.5$ .

(d) This was often correct. Common errors were 2 and 8.

(e) Some correct answers were seen. There were also some octagons of the correct size but wrongly positioned. There were some attempts where construction lines were drawn from  $(-3, -3)$  to the vertices of the octagon but these were taken no further.

(f) (i) There were very few correct answers seen. Most candidates did not relate the octagon given here to the original octagon  $B$  shown on the grid. So it was very rare to see any systematic approach to this calculation.

(ii) Again, very few correct answers seen. The connection with octagon  $B$  was generally not seen.

- (iii) Again, very few correct answers. As before, there was generally no systematic approach to this calculation.

Answers: (a)  $-1$ ; (b) correct triangle; (c)  $x = -2.5$ ; (d) 4; (e) correct octagon; (f)(i) 1575; (ii) 30;  
(iii) 10 350.

### Question 10

- (a) (i)(a) This was usually correct.
- (b) The 'angle at the centre' theorem was usually remembered correctly, so this was often correct. Sometimes the relationship between the angle at the centre and angle at the circumference were reversed leading to an answer of  $x$  here.
- (c) Candidates were usually able to proceed correctly from their answer to part (b).
- (ii) This part was usually attempted, but not often successfully. A common wrong answer was 57. Candidates did not make it clear what angles they were working out.
- (b)(i) The formula for the area of a sector was generally well known. Some candidates went on to calculate the volume of the prism. Some candidates added  $20 \times 8$  to the sector area, perhaps thinking that this was a 2-D shape.
- (ii) There were some completely correct answers. Commonly, for incorrect answers, the number of areas included was incomplete.

Answers: (a)(i)(a)  $2x$ ; (b)  $4x$ ; (c)  $90 - 2x$ ; (ii) 19; (b)(i) 22.3; (ii) 476.

### Question 11

- (a) (i) Generally attempted, with correct and wrong answers in about equal measure. The nature of the graph was well understood. Not all candidates worked out the horizontal scale correctly.
- (ii) The basic principle, time = distance / speed, was understood. This led to many correct answers. Some candidates had difficulty adding 0.25 hours to 12 30. The wrong answer 12 55 was common.
- (iii) The easier scale on the vertical axis led to greater success in this part.
- (iv)(a) Again, dealing accurately with the horizontal axis seemed to be the main problem here. Most candidates understood that a horizontal line was required at  $d = 5.4$ . Many failed to draw it the correct length. Finding the correct end point at (15 39, 0) was also a problem for many candidates.
- (b) There were few correct answers seen here.
- (b)(i) There were many correct pie charts drawn. For many candidates, drawing accurate angles of  $30^\circ$  and  $150^\circ$  was clearly a problem.
- (ii) This part was well understood, with many correct answers.
- (iii) There were some good attempts at this question, some of which were completely successful. The principles were generally understood. A common error was to express some of the probabilities using a denominator of 12, when a denominator of 11 was required.

Answers: (a)(i) 23 to 25; (ii) 12 45; (iii) 1.9; (iv)(a) Straight lines to (14 45, 5.4) and from (14 45, 5.4 )  
to (15 39, 0); (b) 6; (b)(i) Correct sectors and labels; (ii)  $\frac{5}{12}$ ; (iii)  $\frac{41}{66}$ .

# MATHEMATICS D

---

Paper 4024/22

Paper 22

## Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and remember necessary formulae. All working must be clearly shown, and in numerical work, a suitable level of accuracy must be used in any given calculation, and maintained throughout a series of linked calculations.

## General comments

Scripts were seen covering the whole range of marks available. Many of these were well presented. It is helpful when candidates add headings to explain the particular step in a problem that they are working on. This gives clarity and organisation to the solution of problems that contain several stages.

The paper included opportunities for candidates at all levels to show what they could do.

In **Section B, Question 8** was a popular choice question. **Question 7** was not, and was frequently omitted.

In a question such as **10(b)**, where the scales of the axes are specified, candidates should be aware that they are expected to use them. It is also expected that the axes will be drawn so that the origin is their point of intersection. To draw the axes on the margins of the grid creates difficulties for accurate plotting. Some candidates seem unaware that the types of functions that they are asked to plot lead to smooth curves, where it should be immediately clear if a point has been mis-plotted or miscalculated.

## Comments on specific questions

### **Section A**

#### **Question 1**

- (a) (i) This part was generally well done. A common error was to find the percentage of the salary that was taxed. Some candidates used the fraction  $\frac{18750}{5625}$ . The use of the denominator 13125 was also seen.
- (ii) With the need to consider the additional data required for this part, this was clearly more demanding for candidates. Most had no problem calculating 22% of 13125. Many candidates then continued by subtracting this from 13125 instead of 18750. At the next stage, many candidates assumed the data given in the question to be either incorrect or incomplete. As well as variations on the number of weeks in a year, additional 7's and 12's were seen. Occasionally, multiplication by 52 was seen.
- (iii) Candidates either knew how to handle this reverse percentage arithmetically, or tried some calculation using the numbers given. The commonest of these was either 25% or 125% of 18750. Candidates seem unable or unwilling to approach problems of this type by constructing an equation from the data given.
- (b) (i) This part was generally handled well.
- (ii) Another successful part. A few candidates multiplied by 1.53.

- (iii) Both conversion factors were required in this part. Candidates generally understood how to do this. The common errors were to apply only one conversion factor or to apply the reciprocal of the correct factor. A few candidates used the product of the two individual factors.

Answers: (a)(i) 30; (ii) 305.05; (iii) 15 000; (b)(i) 65 407.50; (ii) 294.12; (iii) 877.19.

### Question 2

- (a) (i) Candidates generally got this correct. The common wrong answer was 46. Other wrong answers seen included 67, 113 and 157.
- (ii) At this stage in the question, the only acceptable reason in support of  $90^\circ$  was a reference to the angle in a semicircle. Very few candidates realised this. A common error was to use the properties of parallel lines that had not yet been established. Attempts at references to circle theorems included statements such as the angle at a tangent.
- (iii) There were relatively few absolutely clear written explanations. Probably the most popular one was that both lines were perpendicular to  $AE$ . Many candidates supported their answers by showing equal corresponding angles clearly on their diagram.
- (b) Very few fully argued explanations were seen. A number of candidates used the equal sides of the isosceles triangles and clearly related these to the perimeters in question. There were a number of attempts based on similar triangles, and others attempting to use vectors. This part was often omitted.

Answers: (a)(i) 23; (ii) 90 angle in a semicircle; (iii) Parallel lines established; (b) Convincing argument.

### Question 3

- (a) Generally well done. There were some integer answers given, such as 1 and 13.
- (b) The correct answer was rarely seen. Many candidates isolated the probabilities  $\frac{3}{16}$  and  $\frac{7}{16}$ . This often led to the incorrect answer  $\frac{21}{256}$ . Some candidates added these probabilities.
- (c) (i) Generally well done. A wide variety of other answers were seen, including some involving  $n$  and  $m$ .
- (ii) The correct simultaneous equations were frequently seen. In these cases, full marks were usually obtained. A number of candidates who attempted to eliminate one variable by substitution substituted back into the same equation.
- (d) This part was done reasonably well.

Answers: (a)  $\frac{1}{16}$ ; (b)  $\frac{42}{256}$ ; (c)(i) 26; (ii)  $m = 5$   $n = -3$ ; (d) 17.

### Question 4

- (a) (i) Most candidates calculated a volume. In some cases, this was left as  $3 \times 7 \times 10$ , in other cases, instead of introducing the factor  $\frac{1}{2}$ , the factor  $\frac{1}{3}$  was used. Some solutions involved areas.
- (ii) There were a number of completely correct solutions seen. Most candidates identified the triangle and at least one rectangle. The rectangle requiring the use of Pythagoras' theorem for the dimension not given in the diagram proved to be problematic. There were a number of candidates who thought this was  $\sqrt{7^2 - 3^2}$ , others used 7
- (b) (i) This part was quite well done. Candidates were able to visualise the trigonometry required from the diagram in the question. There were some solutions that used cos instead of sine.

- (ii) Many candidates seemed unable to continue with this part. Many others, who realised that the diagram for this part was largely in place, simply requiring the addition of an appropriate angle, did so successfully.

Answers: (a)(i) 105; (ii) 197.2; (b)(i) 0.845; (ii) 0.280.

### Question 5

- (a) There were some correct solutions. It was generally clear that most candidates had knowledge of the formula for the circumference of a circle. Not all candidates were able to adapt their thinking to use their knowledge effectively in the situation here. Work needs to be set out clearly and fully explained. It was not always clear from candidates' working if they were thinking of a complete circle or a semicircle, or if they were using a diameter or a radius. A common error was the answer 31.8. There were a number of solutions that did not use the expected circle formula. Answers such as 50 and 100 were common in these cases. Some candidates worked with areas at this stage.
- (b) The situation here seemed to be clearer to candidates and this was reflected in the working shown. The basic area results required were well known. The composition of the circular area sometimes lacked the definition described above.
- (c) (i) This part depended on finding the circumference of the outer circle. Quite a number of candidates managed to indicate, in some way, that this was the circle they were working in. A few candidates worked through to a successful conclusion. A number of candidates who tackled this part wanted to use the formula for the arc length. They achieved this by measuring the angle  $AOT$  on the diagram. This solution was not accepted.
- (ii) There were relatively few completely correct answers. Candidates gained credit if their working showed accurate knowledge of the formula for arc length. Most candidates who used an angle value in the previous part managed to show their work was consistent by getting back to it here.

Answers: (a) 63.7; (b) 9549.2; (c)(i) 18.8; (ii) 31.

### Question 6

- (a) Candidates often achieved full marks in this question. A common error involved the angle of  $104^\circ$ . This was often seen as  $76^\circ$ . The idea of the parallel sides was not always used. A common error here was to measure 4.5 cm from  $A$  to  $D$ , and 4.5 cm from  $B$  to  $C$ . A number of candidates omitted this question.
- (b) Most candidates showed appropriate understanding of the idea of scale.

Answers: (a) Correct shape  $ABCD$ ; (b) 115 to 125.

### Section B

#### Question 7

- (a) (i) There were very few complete solutions. Expressing  $DE$  in vector form was well understood.  $BC$  proved to be a problem. Most candidates who attempted this were defeated by the ratios required.
- (ii) The ratio 3 : 5 that eluded most candidates in the previous part was seen here in a minority of cases. Some of these did successfully convert to 9 : 25. A common answer was 2 : 3.
- (b) (i) There was a noticeably better response here. Some candidates offered solutions without calculating the coordinates of the vertices first. These were rarely correct.
- (ii) A variety of transformations were offered here. The correct one was known by a minority of candidates. A common error was to call it an enlargement.
- (iii) There were a number of successful solutions where the given matrices were multiplied in the correct order. Some candidates added the matrices.

- (iv) Candidates usually answer well when asked to find the inverse of a given 2 by 2 matrix. Here only a few candidates realised that this is what this question was about.

Answers: (a)(i) 874; (ii) 9:25; (b)(i) Correct triangle; (ii) Stretch; (iii)  $\begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}$ ; (iv)  $\frac{1}{2}\begin{pmatrix} 1 & 0 \\ -2 & 2 \end{pmatrix}$ .

### Question 8

- (a) (i) The majority of candidates evaluated this correctly. Common incorrect evaluations were 0.672, 0.072 and 0.36.
- (ii) There were a good number of correct answers seen. Most candidates applied an accurate transposition at some stage. Attempts at squaring to remove the square roots were less successful. Quite a number of candidates shortened the square root sign when writing down the initial expression thus changing it, leading to multiplication by  $g$ . Some solutions offered were numerical.
- (b) This was generally well done. As usual, care is needed when there is a minus sign outside a bracket. For some candidates, a common misunderstanding was that  $45 - (p + 3)$  was evaluated as, for example,  $45(p + 3)$ . Some candidates ended up with a quadratic equation.
- (c) Most candidates knew how to combine the fractions, and did so correctly. A common error after this step was to equate the correct numerator to such as 12.
- (d) Well answered by the majority of candidates.

Answers: (a)(i) 2.24; (ii)  $\frac{T^2 g}{4\pi^2}$ ; (b) 14; (c) -5.5; (d) -0.41 -3.26.

### Question 9

- (a) (i) The area sine formula for the triangle is well known. Here, the main test was to evaluate the area to 3 decimal places. Many candidates just quoted the formula and the 11.05 given in the question.
- (ii) This part was generally well done. Suitable accuracy is always required.
- (iii) Candidates did not always find this straightforward. Many equations seen were spoilt by an incorrect use, or omission, of the factor  $\frac{1}{2}$ . Candidates revealed some confusion about the interpretation of the obtuse angle. There were some solutions using the alternative method finding the height of the parallelogram. Since this generally involved the extra step of evaluating the height, solutions usually were spoilt by premature approximation.
- (b) (i) The cosine rule was well known. This question was well done with more correct evaluations than not.
- (ii) Most candidates adapted easily to the angle form of the cosine rule. Many candidates did not subtract  $30^\circ$ .

Answers: (a)(i) 11.046...; (ii) 39.1; (iii) 136; (b)(i) 6.16; (ii) 41.4.



### Question 10

- (a) Usually correct. The common error was 3 for  $x = -2$ .
- (b) Since the scales required were specified in the question, full marks depended on their use. Many candidates preferred their own, usually using 1 cm to 1 unit on the horizontal axis.
- (c) Most candidates understood what was required here. The tangents offered were not always well drawn. The calculation of the gradient was not always reliable.
- (d)(i) Usually accurate.
- (ii) The demand of this question was largely understood. There were many correct answers. A number of candidates who clearly understood the essentials of what was being asked stated values slightly outside the expected values.
- (e) It was expected that a comparison of the expressions in the stem of the question and part (e) would lead to the use of the line  $y = -3$  in conjunction with the graph drawn in part (b). This method was seen in a number of cases. Some candidates drew another graph to represent the new function in this part. Many candidates either used the formula again, or omitted this part.

Answers: (a) 11 11; (b) Correct scales and curve; (c) 2; (d)(i) -5; (ii)  $a = -1$   $b = 5$ ; (e) 0.6 3.4.

### Question 11

- (a) Not many fully correct histograms were seen. Many candidates managed only the first or second column correctly.
- (b)(i) This part was generally well done. There were a few candidates who plotted values at mid-interval points. Generally, the given scales were negotiated successfully.
- (ii)(a) The idea of the median value was well understood. A number of candidates based their mid-value on the upper limit of the graph, 250, rather than the number of potatoes in the distribution.
- (b) This part was generally well done. Some candidates were unclear that the range required is found by reading off the values of the upper and lower quartiles and finding their difference. A number of  $\frac{3}{4} \times 240 - \frac{1}{4} \times 240$  were seen.
- (iii) A common error here was to use values obtained from the cumulative frequency curve rather than the values that could be obtained from the table in part (b).
- (iv)(a) Generally well done. Some candidates recovered from the error described in the previous part.
- (b) There was a poor response to this part. When not omitted, a common error was the lack of subtraction from 240.

Answers: (a) Correct histogram; (b)(i) Correct cumulative frequency curve; (ii)(a) 195; (b) 72 to 88; (iii) 50 78 72 32 4; (iv)(a) 36; (b) 8.