## Cambridge International Examinations

## Cambridge Ordinary Level



## STATISTICS

4040/22
Paper 2
October/November 2014
2 hours 15 minutes
Candidates answer on the question paper.
Additional Materials: Pair of compasses
Protractor

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.
Answer all questions in Section $A$ and not more than four questions from Section B.
If working is needed for any question it must be shown below that question.
The use of an electronic calculator is expected in this paper.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

## Section A [36 marks]

## Answer all of the questions 1 to 6 .

1 (i) Explain the meaning of the term 'discrete variable'.
$\qquad$
$\qquad$
(ii) Give an example of a continuous variable.
$\qquad$
(iii) Explain the meaning of the term 'qualitative variable'.
$\qquad$
$\qquad$
(iv) Give an example of a discrete quantitative variable.
$\qquad$

2 The number of people living in each home on a street was counted.

| Number of <br> people | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 or <br> more |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| Number of <br> homes | 1 | 2 | 3 | 3 | 5 | 4 | 8 | 3 | 0 |

For example, 3 homes have 2 people living in them.
(i) State the modal number of people living in a home on this street.
$\qquad$
(ii) Find the median number of people living in a home on this street.

It was later discovered that an error had been made, and that $h$ homes with 8 or more people were missing from the original data.
(iii) Find the maximum possible value of $h$ such that the median will be unchanged when the extra data is included.

3 Maria and Nico each have a tin containing 5 white, 8 milk and 7 dark chocolates.
(i) Maria selects two chocolates at random from her tin and eats them.

Find the probability that
(a) both are white chocolates,
(b) exactly one is a white chocolate.
(ii) Nico selects chocolates at random from his tin until he finds a milk chocolate. He returns unwanted chocolates to the tin after each selection.

Find the probability that it will take him fewer than 3 attempts to find a milk chocolate.

4 The masses, in grams, of a sample of potatoes from a crop are shown in the table below.

| Mass, $m(\mathrm{~g})$ | Number of potatoes |
| :---: | :---: |
| $30 \leqslant m<50$ | 14 |
| $50 \leqslant m<100$ | 63 |
| $100 \leqslant m<150$ | 82 |
| $150 \leqslant m<250$ | 47 |
| $250 \leqslant m<400$ | 19 |
| $400 \leqslant m<600$ | 12 |

(i) For these data, state the name of the most appropriate measure of central tendency and the name of the most appropriate measure of dispersion. Give a reason for your answers.

Measure of central tendency $\qquad$
Measure of dispersion $\qquad$
Reason $\qquad$

Potatoes over 300 g are classified as 'large'.
(ii) Without drawing a graph, calculate an estimate of the number of potatoes from this sample which are classified as 'large'.

5 At a school the 100 male and 120 female students choose to study one of the three options Music, Drama or Art.
Their choices are illustrated in the chart below.

(i) State the full name for this type of chart.
$\qquad$
(ii) Calculate the numbers of males and females taking each option and insert them into the table below.

|  | Music | Drama | Art |
| :--- | :--- | :--- | :--- |
| Male |  |  |  |
| Female |  |  |  |

(iii) Display your data from part (ii) in a fully-labelled dual bar chart using the key provided.

(iv) Give one advantage that the dual bar chart you have drawn has over the chart given at the start of the question.
$\qquad$
$\qquad$

6 Two unbiased six-sided dice, one blue and one green, each with faces numbered 1, 2, 3, 4, 5 and 6 are thrown.
The following are some of the possible outcomes.
A A 6 is obtained on the blue die
$B \quad$ An even number is obtained on the green die
C An odd number is obtained on the green die
D A 4 is obtained on the green die
(i) From the list above, state all the pairs of
(a) independent events,
$\qquad$
(b) mutually exclusive events.
$\qquad$
(ii) Find $\mathrm{P}(A \cup B)$.

Event $D$ and a fifth event, $E$, are known to be mutually exclusive.
(iii) Find the smallest and largest possible values for $\mathrm{P}(E)$.

## Section B [64 marks]

## Answer not more than four of the questions 7 to 11 .

Each question in this section carries 16 marks.

7 (a) The following is a time series graph for a variable $V$.

(i) State two purposes of finding moving average values.

1 $\qquad$
$\qquad$

2 $\qquad$
$\qquad$
(ii) If $n$-point moving average values were to be calculated for the variable $V$, state an appropriate value for $n$.
(iii) State whether or not it would be necessary to centre the moving average values in this case. Clearly explain the reason for your answer.
$\qquad$
$\qquad$
$\qquad$
(b) The table below shows fertilizer sales (in thousands of tonnes) by a company each quarter for a period of 3 years.

(i) Calculate the values of $a, b$ and $c$ and enter them in the table above.
(ii) Use the 'Sales' and 'Centred 4-point moving average' values for quarter II of 2011 and 2012 to find an estimate for the seasonal component of quarter II. Give your answer, in thousands of tonnes, correct to one decimal place.
[3]
(iii) Plot the centred moving average values on the grid below and draw the trend line.

(iv) Use your trend line and answer to part (ii) to estimate the sales for quarter II of 2013.

8 In order to calculate a weighted aggregate cost index, a café owner divides his expenditure into three categories: Ingredients, Electricity and Wages.
He collects the following data for the year 2011.
Ingredients cost a total of $\$ 15600$.
Electricity cost $\$ 0.09$ per unit.
A total of 5000 units of electricity were used.
A total of 4000 staff hours were worked.
The average wage per hour for all the staff was $\$ 6.50$.
(i) Show that the café owner should assign weights to the three categories Ingredients, Electricity and Wages in the ratio 312:9:520.
(ii) Using the following information, complete the table below, giving price relatives to the nearest integer where appropriate.

2011 is the base year.
The cost of ingredients increased by 8\% from 2011 to 2012.
The price of electricity rose to $\$ 0.11$ per unit in 2012.
The average wage per hour for all staff fell by 3\% from 2011 to 2012.

|  | Price relatives |  |
| :--- | :---: | :---: |
|  | 2011 | 2012 |
| Ingredients |  |  |
| Electricity |  |  |
| Wages |  |  |

(iii) Calculate a weighted aggregate cost index for the year 2012, taking 2011 as base year, giving your answer correct to one decimal place.
(iv) Use the index calculated in part (iii) and the costs for 2011 to estimate, to 3 significant figures, the total cost of running the café in 2012.
(v) Give two possible reasons why your estimate for 2012 may be very inaccurate.

Reason 1 $\qquad$
$\qquad$
Reason 2 $\qquad$
$\qquad$

## 9 In this question give all probabilities as exact fractions.

A turn at a game consists of throwing a pair of unbiased coins, each with a head on one side and a tail on the other. A point is scored every time a turn produces a pair of heads. A game consists of three turns.
(i) If one game is played,
(a) show that the probability of scoring three points is $\frac{1}{64}$,
(b) find the probability of scoring exactly two points.
(ii) If $X$ is the number of points scored in one game, find the probability of each of the remaining possible values of $X$. Hence produce a table showing all the possible values of $X$ together with their probabilities.

Each game of three turns costs $\$ 4$ to play.
A player wins $\$ 58$ for scoring three points.
A player wins nothing for scoring fewer than two points.
Rashid decides to play the game, and scores exactly two points.
(iii) Find how much money he would win, assuming it is a fair game.

As an alternative to taking the money won in part (iii), a player who has scored exactly two points is given the option of throwing another single coin once. This coin is weighted in favour of tails, and is four times more likely to show a tail than a head.

If this option is taken the player will win $\$ 50$ for a head and $\$ 12.50$ for a tail.
(iv) Determine, by calculation, whether or not Rashid should risk throwing the extra coin.

10 (a) The number of calls per day received at a fire station over a period of time is shown in the table below.

| Number of calls per day | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Number of days | 13 | 11 | 7 | 6 | 4 | 1 |

For example, 3 calls were received on 6 of the days.
(i) Calculate how many calls were received, in total, over the period of time.
$\qquad$
(ii) Calculate the mean number of calls per day, correct to one decimal place.
(b) The mean and standard deviation of three numbers $a, b$ and $c$ are 11 and 3 respectively.

Complete the table below by finding the mean and standard deviation of each of the four sets of numbers shown.

|  | Mean | Standard deviation |
| :--- | :--- | :--- |
| The three numbers |  |  |
| $a-1, b-1, c-1$ |  |  |
| The three numbers |  |  |
| $\frac{a}{2}, \frac{b}{2}, \frac{c}{2}$ |  |  |
| The three numbers <br> $5 a+3,5 b+3,5 c+3$ |  |  |
| The six numbers <br> $a, a, b, b, c, c$ |  |  |

(c) The students in a mathematics class were given a test in Algebra and a test in Geometry, both with a maximum mark of 100. The following table summarises their results.

|  | Class mean | Class standard <br> deviation |
| :---: | :---: | :---: |
| Algebra | 55 | 10 |
| Geometry | 40 | 4.5 |

(i) Explain what these figures tell you about the differences between the marks in the Algebra test and the marks in the Geometry test.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(ii) Priyanka scored 65 in her Algebra test and 49 in her Geometry test.

Use the class means and standard deviations to state, with a reason, in which test Priyanka scored better in relation to the rest of the class.
$\qquad$
$\qquad$
$\qquad$
(iii) The highest mark scored by any pupil in the Algebra test was 87. It is required to scale the marks so that the scaled mean is 60 and the scaled highest mark is 100.

Calculate the scaled standard deviation which must be used to achieve this.

11 At a jam-making factory, 90 jars are filled with jam in fifteen minutes. A sample of 9 of the jars needs to be taken to check that the mass of jam in the jars is within acceptable limits.
The jars are numbered from 00 to 89 .
Asad thinks that the best method for selecting the sample is to take a simple random sample.

## RANDOM NUMBER TABLE

| 47 | 00 | 51 | 96 | 32 | 47 | 85 | 11 | 67 | 05 | 10 | 90 | 28 | 73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 92 | 01 | 55 | 83 | 76 | 34 | 41 | 29 | 07 | 24 | 63 | 15 | 59 | 81 |
| 44 | 03 | 59 | 99 | 14 | 27 | 20 | 30 | 09 | 78 | 60 | 04 | 81 | 65 |

(i) Starting at the beginning of the first row of the random number table, and working along the row, find Asad's sample, ensuring that no jar is selected more than once.
$\qquad$
Omar thinks that the best method for selecting the sample is to take a systematic sample.
(ii) By starting at the beginning of the second row of the random number table, and working along the row, select the first jar in Omar's sample. State also the numbers of the remaining jars in his sample.

The jam-making factory has three machines $\mathrm{A}, \mathrm{B}$ and C which put jam into jars, and two packers $X$ and $Y$ who pack jars into boxes.
Each jar is filled by one of the machines and then packed into a box by one of the packers.
The two-way table shows how many jars filled by each machine were packed by each packer in fifteen minutes.

|  | Machine $A$ | Machine B | Machine C |
| :---: | :---: | :---: | :---: |
| Packer $X$ | 10 | 11 | 18 |
| Packer $Y$ | 10 | 19 | 22 |

(iii) If a sample of size 9 stratified by machine were to be taken, calculate how many jars from each machine would be required.

Machine A $\qquad$
Machine B $\qquad$
Machine C

The jars from machines A, B and C are those numbered 00-19, 20-49 and 50-89 respectively.
(iv) Comment on how accurately the samples taken by Asad and Omar represent the jars filled by each machine.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(v) Starting at the beginning of the third row of the table, and moving along the row, select a sample of size 9 stratified by machine. Use every number if the machine to which it relates has not yet been fully sampled.
$\qquad$
$\qquad$
(vi) If a sample of size 9 stratified by packer were to be taken, calculate how many jars from each packer would be required.

Packer X
Packer $Y$
(vii) If a stratified sample were to be taken, state whether it would be more appropriate, in this case, to stratify by machine or by packer. Explain your answer.
$\qquad$
$\qquad$
$\qquad$

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