CAMBRIDGE
INTERNATIONAL EXAMINATIONS

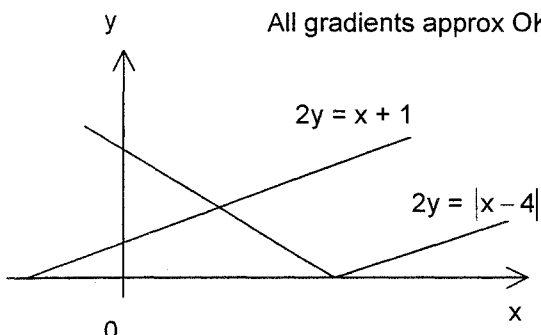**NOVEMBER 2001**

ADVANCED SUBSIDIARY LEVEL

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT : 8709/2

MATHEMATICS

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	$2\sec^2x - \tan x = 5$ Use of $\sec^2x = 1 + \tan^2x$	M1	Use of tan – sec link
	$\rightarrow 2\tan^2x - \tan x - 3 = 0$	A1	Correct only
	Solution of this	DM1	Correct attempt to solve
	$\tan x = -1$ or 1.5		
	$x = 135°$ or $315°$ or $56.3°$ or $236.3°$	A1A1√	A1 for one pair correct. A1sq for other pair.
		5	

2	(i)	$4^x = u^2$ and $2^{x+1} = 2u$	B1	For both values
		$u^2 = 2u + 12$		
	(ii)	Leads to $u = 4.6055$ (or $1 + \sqrt{13}$)	B1	For correct value of u – even if other given
		Solution of $2^x =$ "his value" by logs	M1	Realises need to use logs (or TI if accurate)
		$x = \log 4.6055 \div \log 2$	M1	$\log \div \log$
		$x = 2.20$	A1	Co to 3 sig figs (but allow 2.2)
				(Loses this A mark if 2 answers given)
			5	

3	(i)	Graph of $2y = x + 1$	B1	Approx correct – no values needed
		Graph of $2y = |x - 4|$	M1	Must be V-shape – no negatives – to x-axis
		At (2,0)		
		All gradients approx OK	A1	Two approx parallel, other with negative m

	(ii)	Solution occurs when $2y = x + 1$ and	M1	Recognition of where solution lies
		$2y = 4 - x$	M1	Must be using $(4 - x)$ not $(x - 4)$
		$x = 1.5$, $y = 1.25$	A1	Both needed
			6	

4	Attempt at $Y = mX + c$	**M1**	Attempt at any $y = mx + c$ eqn
	$Y = -0.6X + c$	**A1**	m and c correct
	Puts $Y = \ln y$ and $X = \ln x$	**M1**	Putting $Y = \ln y$ and $X = \ln x$
	$\ln y = -0.6\ln x + 3$		
	$y = e^3 x^{-0.6}$	**M1**	Correct elimination of logs
	$n = -0.6$ and $A = e^3 = 20.1$	**A1A1**	
		6	

5	(a)	$y = \dfrac{e^{2x}}{2x+3}$ $\quad dy/dx = \dfrac{(2x+3)2e^{2x} - e^{2x}.2}{(2x+3)^2}$	**M1**	Correct u/v formula – or uv with $e^{2x}(2x+3)^{-1}$
			A1	Correct unsimplified
		If $x = 0$, $dy/dx = 4/9$	**A1**	Co
	(b)	Implicit differentiation.	**M1**	Some evidence of implicit needed
		$2x + 2y\,dy/dx = y + x\,dy/dx$	**A1A1**	A1 LHS, A1 RHS
		At (3,2), $dy/dx = -4$		
		Eqn of tangent $\quad y - 2 = -4(x-3)$	**M1**	Must have used calculus, not for normal
		or $y + 4x = 14$	**A1**	Any form ok.
			8	

6	(i)	$y = x^2\cos x \quad dy/dx = 2x\cos x - x^2\sin x$	**M1**	Correct uv formula
			A1	Unsimplified ok
		$= 0$ when $x = 0$ or $2\cos x = x\sin x$	**M1**	Putting his $dy/dx = 0$
		$\rightarrow x\tan x = 2.$	**A1**	Co
	(ii)	$u_2 = 1.107 \quad u_3 = 1.065 \quad u_4 = 1.081$	**M1**	Correct manipulation of u_{n+1} from u_n
		$u_5 = 1.075 \quad u_6 = 1.078 \quad u_7 = 1.077$	**A1**	First two correct
		$\rightarrow$ Limit of 1.08	**A1**	Correct limit
	(iii)	Since a limit is reached $(=L)$		
		$u_{n+1} = u_n = L$		
		$L = \tan^{-1}(2/L)$	**M1**	Putting $u_{n+1} = u_n = L$
		$L\tan L = 2.$	**A1**	Co
			9	

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(i)

$$\int_0^{\frac{\pi}{4}} \sin 2x\, dx = \left[\frac{-\cos 2x}{2}\right] = 0 - (-\tfrac{1}{2}) = \tfrac{1}{2}$$

M1 — Needs "–" and cos 2x.

A1 — Co

$$\int_0^{\frac{\pi}{4}} \cos^2 x\, dx = \int \frac{\cos 2x}{2} + \frac{1}{2}\, dx$$

M1 — Using double angles + attempt at integration

$$= \left[\frac{\sin 2x}{4} + \frac{x}{2}\right]$$

A1 — Co

DM1 — Use of limits 0 to π/4

$$= \frac{1}{8}(2 + \pi)$$

A1 — Co beware of fortuitous answers.

(ii)

$$\int (2s + 3c)^2\, dx = \int (4s^2 + 9c^2 + 12sc)\, dx$$

B1 — Correct squaring – needs all terms

$$12sc = 6\sin 2x \quad \text{Integral} = 6 \times \frac{1}{2} = 3$$

B1 — There could be alternatives to these marks.

$$9c^2 \quad\quad \text{Integral} = 9 \times \frac{1}{8} \times (\pi + 2)$$

B1 — They could also be implied.

$$4s^2 = 4 - 4c^2$$

$$\text{Integral} = 4x \text{ between } 0 \text{ and } \frac{1}{4}\pi$$

M1 — Dealing correctly with ∫4s²

$$4 \times \text{integral of } c^2 \text{ from } 0 \text{ to } \frac{1}{4}\pi$$

$$= 9.36 \text{ or } 13\pi/8 + 17/4$$

A1 — Correct in either form.

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