

**JUNE 2002**

**GCE Advanced Level  
GCE Advanced Subsidiary Level**

**MARK SCHEME**

**MAXIMUM MARK : 75**

**SYLLABUS/COMPONENT : 9709 /3, 8719 /3**

**MATHEMATICS  
(Pure 3)**



Page 1	Mark Scheme	Syllabus	Paper
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- 1 *EITHER:* Express LHS in terms of  $\cos\theta$  and  $\sin\theta$  or in terms of  $\tan\theta$  M1  
 Make sufficient relevant use of double-angle formula(e) M1  
 Complete proof of the result A1  
*OR:* Express RHS in terms of  $\cos\theta$  and  $\sin\theta$  or in terms of  $\tan\theta$  M1  
 Express RHS as the difference (or sum) of two fractions M1  
 Complete proof of the result A1 **3**
- [SR: an attempt ending with  $\frac{1 \cdot \tan^2\theta}{\tan\theta} = \cot\theta - \tan\theta$  earns M1 B1 only.]
- 2 *EITHER:* Show correct (unsimplified) version of the  $x$  or the  $x^2$  or the  $x^3$  term M1  
 Obtain correct first two terms  $1 + x$  A1  
 Obtain correct quadratic term  $2x^2$  A1  
 Obtain correct cubic term  $\frac{14}{3}x^3$  (allow  $\frac{28}{6}$ , 4.67, 4.66 for the coefficient) A1  
 [The M mark may be implied by correct simplified terms, if no working is shown. It is not earned by unexpanded binomial coefficients involving  $-\frac{1}{3}$ , e.g.  ${}^{-\frac{1}{3}}C_1$  or  $\binom{-\frac{1}{3}}{2}$ .]  
 [An attempt to divide 1 by the expansion of  $(1 - 3x)^{\frac{1}{3}}$  earns M1 if the expansion has a correct (unsimplified)  $x$ ,  $x^2$ , or  $x^3$  term and if the partial quotient contains a term in  $x$ . The remaining A marks are awarded as above.]
- OR:* Differentiate and calculate  $f(0)$ ,  $f'(0)$ , where  $f(x) = k(1 - 3x)^{-\frac{1}{3}}$  M1  
 Obtain correct first two terms  $1 + x$  A1  
 Obtain correct quadratic term  $2x^2$  A1  
 Obtain correct cubic term  $\frac{14}{3}x^3$  (allow  $\frac{28}{6}$ , 4.67, 4.66 for the coefficient) A1 **4**
- 3 Attempt to find  $a$  and/or quadratic factor by division or by inspection M1  
 Obtain partial quotient or factor  $x^2 - x$  A1  
 State answer  $a = 6$  B1  
 State or imply the other factor is  $x^2 - x + 3$  A1 **4**
- [The M1 is earned if division has produced a partial quotient  $x^2 + bx$ , or if inspection has an unknown factor  $x^2 + bx + c$  and has reached an equation in  $b$  and/or  $c$ .]  
 [SR: a correct division with unresolved constant remainder can earn M1A1B0A1.]  
 [NB: successive division by a pair of incorrect linear factors, e.g.  $x - 1$  and  $x + 2$  or  $x + 1$  and  $x + 2$ , can earn M1A0 or M1A1(if their product is of the form  $x^2 + x + k$ ).]

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- 4 (i) Use the formula correctly at least once  
 State  $\alpha = 1.26$  as final answer  
 Show sufficient iterations to justify  $\alpha = 1.26$  to 2d.p., or show there is a sign change in the interval (1.255, 1.265)
- (ii) State any suitable equation in one unknown e.g.  $x = \frac{2}{3} \left( x + \frac{1}{x^2} \right)$   
 State exact value of  $\alpha$  (or  $x$ ) is  $\sqrt[3]{2}$  or  $2^{\frac{1}{3}}$
- 5 Obtain derivative  $\pm 2\sin x + k \cos 2x$  or  $\pm 2\sin x + k(\cos^2 x \pm \sin^2 x)$   
 Equate derivative to zero and use trig formula to obtain an equation involving only one trig function  
 Obtain a correct equation of this type e.g.  $2\sin^2 x + \sin x - 1 = 0$  or  $\cos 2x = \cos \left( \frac{1}{2} \pi - x \right)$   
 Obtain value  $x = \frac{1}{6} \pi$  (allow 0.524 radians or  $30^\circ$ )  
 Show by any method that the corresponding point is a maximum point  
 Obtain second value  $x = \frac{5}{6} \pi$  (allow 2.62 radians or  $150^\circ$ ), and no others in range  
 Determine that it corresponds to a minimum point
- 6 (i) State or imply  $f(x) = \frac{A}{(3x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)}$   
 State or obtain  $A = -3$   
 State or obtain  $B = 2$   
 Use any relevant method to find  $C$   
 Obtain  $C = 1$   
 [Special case: allow the form  $\frac{A}{(3x+1)} + \frac{Dx+E}{(x+1)^2}$  and apply the above scheme ( $A = -3, D = 1, E = 3$ ).]  
 {SR: if  $f(x)$  is given an incomplete form of partial fractions, give B1 for a form equivalent to the omission of  $C$ , or  $E$ , or  $B$  in the above, and M1 for finding one coefficient.}
- (ii) Integrate and obtain terms  $-\ln(3x+1) - \frac{2}{(x+1)} + \ln(x+1)$   
 Use limits correctly  
 Obtain the given answer correctly

M1  
 A1  
 A1 3  
 B1  
 B1 2  
 M1  
 M1  
 A1  
 A1  
 A1  
 A1  
 A1 7  
 B1  
 B1  
 B1  
 M1  
 A1 5  
 B1 + B1 + B1 ✓  
 M1  
 A1 5

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- 7 (i) State that  $\frac{dm}{dt} = k(50 - m)^2$  B1  
 Justify  $k = 0.002$  B1 2
- (ii) Separate variables and attempt to integrate  $\frac{1}{(50 - m)^2}$  M1  
 Obtain  $\pm \frac{1}{(50 - m)}$  and  $0.002t$ , or equivalent A1  
 Evaluate a constant or use limits  $t = 0, m = 0$  M1  
 Obtain any correct form of solution e.g.  $\frac{1}{(50 - m)} = 0.002t + \frac{1}{50}$  A1  
 Obtain given answer correctly A1 5
- (iii) Obtain answer  $m = 25$  when  $t = 10$  B1  
 Obtain answer  $t = 90$  when  $m = 45$  B1 2
- (iv) State that  $m$  approaches 50 B1 1
- 8 (i) State or imply a simplified direction vector of  $l$  is  $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ , or equivalent B1  
 State equation of  $l$  is  $\mathbf{r} = \mathbf{i} + \mathbf{k} + \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ , or  $\frac{x-1}{3} = \frac{y}{-1} = \frac{z-1}{2}$ , or equivalent B1 ✓  
 Substitute in equation of  $p$  and solve for  $\lambda$ , or one of  $x, y$ , or  $z$  M1  
 Obtain point of intersection  $-2\mathbf{i} + \mathbf{j} - \mathbf{k}$  A1 4  
 [Any notation is acceptable.]
- (ii) State or imply a normal vector of  $p$  is  $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$  B1  
 EITHER: Use scalar product to obtain  $a + 3b - 2c = 0$  M1  
 Use points on  $l$  to obtain two equations in  $a, b, c$  e.g.  $a + c = 1, 4a - b + 3c = 1$  B1 ✓  
 Solve simultaneous equations, obtaining one unknown M1  
 Obtain one correct unknown e.g.  $a = -\frac{2}{3}$  A1  
 Obtain the other unknowns e.g.  $b = \frac{4}{3}, c = \frac{5}{3}$  A1
- OR: Use scalar product to obtain  $a + 3b - 2c = 0$  M1  
 Use scalar product to obtain  $3a - b + 2c = 0$  B1 ✓  
 Solve simultaneous equations to obtain one ratio e.g.  $a : b$  M1  
 Obtain  $a : b : c = 2 : -4 : -5$ , or equivalent A1  
 Obtain  $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$  A1
- [NB: candidates may transfer from the EITHER to OR scheme by subtracting the two "point" equations, or transfer from OR to EITHER by finding one of the "point" equations.]
- OR: Calculate the vector product  $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$  M1  
 Obtain answer  $-4\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$ , or equivalent A1 ✓  
 Substitute in  $-4x + 8y + 10z = d$  to find  $d$ , or equivalent M1  
 Obtain  $d = 6$ , or equivalent A1  
 Obtain  $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$  A1
- OR: State or imply a correct equation of the plane e.g.  $\mathbf{r} = \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) + \mathbf{i} + \mathbf{k}$  M1  
 State 3 equations in  $x, y, z, \lambda$ , and  $\mu$ , e.g.  $x = 3\lambda + \mu + 1, y = -\lambda + 3\mu, z = 2\lambda - 2\mu + 1$  A1 ✓  
 Eliminate  $\lambda$  and  $\mu$  M1  
 Obtain equation  $-4x + 8y + 10z = 6$ , or equivalent A1  
 Obtain  $a = -\frac{2}{3}, b = \frac{4}{3}, c = \frac{5}{3}$  A1 6
- [SR: condone the use of  $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  for  $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$  in the EITHER scheme and the first OR scheme.]

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- 9 (i) State or imply that  $r = 2$  B1  
State or imply that  $\theta = \frac{1}{3}\pi$  (allow 1.05 radians or  $60^\circ$ ) B1  
Obtain modulus 4, and argument  $\frac{2}{3}\pi$  of  $u^2$  (allow  $2^2$ ; 2.09 or 2.10 radians or  $120^\circ$ ) B1 + B1✓  
Obtain modulus 8 and argument  $\pi$  of  $u^3$  (allow  $2^3$ ; 3.14 or 3.15 radians or  $180^\circ$ ) B1✓ 5  
[Follow through on wrong  $r$  and  $\theta$ ]  
[SR: if  $u^2$  and  $u^3$  are only given in polar form, give B1✓ for  $u^2$  and B1✓ for  $u^3$ .]
- (ii) EITHER: Deduce that  $u^2 - 2u + 4 = 0$  from  $u^3 + 8 = 0$   
OR: Verify that  $u^2 - 2u + 4 = 0$  by calculation B1  
State that the other root is  $1 - i\sqrt{3}$ , or equivalent B1 2  
[NB: stating that the roots are  $1 \pm i\sqrt{3}$  is sufficient for both B marks.]
- (iii) Show both points correctly on an Argand diagram B1  
Show the correct relevant circle B1  
Show line (segment) correctly B1  
Shade the correct region B1 4  
[SR: allow work on separate diagrams to be eligible for the first three B marks.]
- 10 (i) State at any stage that the  $x$ -coordinate of  $A$  is equal to 1, or that  $A$  is the point (1,0) B1 1  
(ii) State  $f'(x) = 2 \frac{\ln x}{x}$ , or equivalent B1  
Use product or quotient rule for the next differentiation M1  
Obtain  $2 \cdot \frac{1}{x} \cdot \frac{1}{x} + 2 \ln x \cdot \left(\frac{-1}{x^2}\right)$ , or any equivalent correct unsimplified form A1  
Verify that  $f''(e) = 0$  A1 4  
(iii) State or imply area is  $\int_1^e (\ln x)^2 dx$  B1  
Use  $\frac{dx}{du} = e^u$ , or equivalent, in substituting for  $x$  throughout M1  
Obtain given answer correctly (allow change of limits to be done mentally) A1 3  
(iv) Attempt the first integration by parts, going the correct way M1  
Obtain  $(u^2 - 2u \pm 2)e^u$ , or equivalent, after two applications of the rule A1  
Obtain exact answer in terms of  $e$ , in any correct form, e.g.  $(e - 2e + 2e) - 2$ , or  $e - 2$  A1 3
- [The substitution in (iii) may be done in reverse i.e. starting with the  $u$  integral and obtaining the  $x$  integral. The M1A1 scheme applies, but only an explicit statement will earn the B1.]  
[The M1A1A1 in (iv) applies to those working in terms of  $x$  and obtaining  $x((\ln x)^2 - 2 \ln x \pm 2)$ , or equivalent.]