THINKING SKILLS **9694/31**

Paper 3 Problem Analysis and Solution

May/June 2011

1 hour and 30 minutes

Additional Materials: Answer Booklet/Paper
Electronic Calculator

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES
Calculators should be used where appropriate.

Answer **all** the questions.
Start each question on a new answer sheet.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question.
The total of the marks for this paper is 50.

This document consists of **8** printed pages and **4** blank pages.

UNIVERSITY *of* CAMBRIDGE
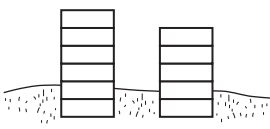International Examinations

[Turn over

1 *Study the information below and answer the questions. Show your working.*
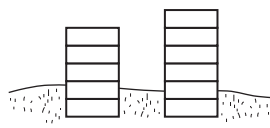
The ruins of old Illyrian temples in the featureless plains of Bolandia have been used as a source of stone blocks for building houses over the years, so the various photographs that have been made over time show columns of different heights. As it's usually cloudy, there are no shadows to help to determine from which direction any particular picture has been taken. All temples originally had columns that were made from six blocks, and all the blocks were identical cylinders, one cubit in height. No blocks have been added to the temples since they were originally built.
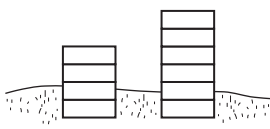
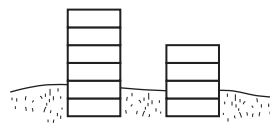One temple has four columns, and this is a plan view showing the heights (in cubits) of the columns now:

This is written as
$$\begin{matrix} 5 & 6 \\ 2 & 4 \end{matrix},$$
with the north shown at the top.

From the north we see which we write as **6 5**.

From the south we see and write **5 6**.
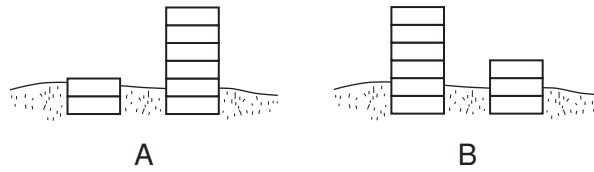
From the east we see and write **4 6**.

From the west we see and write **6 4**.

(a) A different temple of four columns also has the heights **5 6** as seen from one side and **4 6** from another.

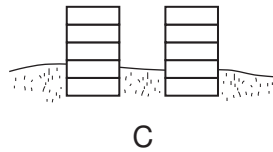**(i)** What is the greatest possible total number of blocks in this ruin? [1]

(ii) What is the smallest possible total number of blocks in this ruin? [1]

(b) Two pictures of another four-column ruin have been found.

A   B

(i) Draw a possible plan view of the ruin to show that these pictures could have been taken at the same time. [1]

A further picture of the same ruin was found.

C

From this it is now certain that the three pictures A, B and C were all taken at different times.

(ii) Give the order in which they were taken, and draw a possible sequence of plan views showing the ruin at the three times. [4]

(c) A larger ruin had nine columns arranged in a 3 by 3 square. Originally they were all 6 cubits high. From one side it now appears as **5 4 5** and from another as **4 2 5**.

(i) What is the greatest number of blocks that can be there now? [1]

(ii) What is the greatest number of blocks that can have been taken away? [2]

2 *Study the information below and answer the questions. Show your working.*

The driving licences issued in Great Britain up until 1 April 1999 did not have a photograph, but there were features to help the police to check if a licence they were shown was likely to be a valid licence for a particular driver.

Jeremy noticed that the six-digit number (shown in bold) on his driving licence might be somehow associated with his date of birth: SMITH **704309** J99RX. He was born on 30 April 1979.

Iain's number is 806210, and he was born on 21 June 1980. Between them they thought they understood how the digits were selected and arranged, and correctly predicted Fred's six-digit number, knowing that he was born on 17 March 1981.

(a) (i) What was Fred's number? [1]

   (ii) Although they could make this prediction knowing the numbers and dates of birth of both Iain and Jeremy, they could not be sure how the numbers were constructed by just looking at the number and date of birth of **only one** of them. Why not? [1]

   (iii) Give an example of a date of birth which would have been sufficient on its own to make this prediction with confidence. [1]

Emma pointed out that it must be a more complicated system than Jeremy thought, as her number is 662126, and her (female) friend Jocelyn has 752232.

Jeremy, knowing that Emma's birthday is 12 December, correctly suggested that this is because a specific number was added to one of the digits for females.

(b) (i) How much is added, and to which digit? [2]

   (ii) What is Jocelyn's date of birth? [1]

(c) Although never implemented, the authorities considered identifying people who had been born outside Great Britain by using a similar system to that which identifies gender.

   Give an example of how this could have been done, within the six digits, without losing any of the existing information. [1]

(d) Sometimes people tried to use the driving licence of one of their parents.

   Given that a police officer can estimate a person's age to within ten years, what is the chance that the deception would be noticed from looking at the person and the number on the driving licence? [1]

(e) Using a random number for making a fake licence for a male, what is the probability that it would fail to give a valid month and date (ignoring the year)? [2]

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[Question 3 is printed on the next page]

3 *Study the information below and answer the questions. Show your working.*

Large blocks of stone can be moved by 'rolling' them. The diagram below shows how a single stone can be moved in this way.

one 360° revolution

start   finish
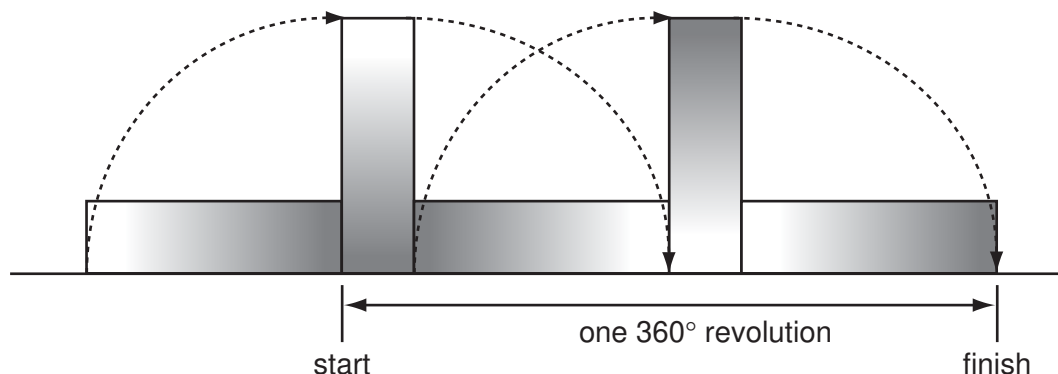
It is quicker to lower a tall stone than to raise it. From experience, a stonemason knows that he can turn a stone through 90° in (b/h) minutes, where b is the length of the face that is flat on the ground, and h is the vertical height of the stone as he is about to turn it.

For example, a block that has two of its dimensions as 5 metres and 2 metres can be turned 180° in 5/2 minutes + 2/5 minutes = 2 minutes 54 seconds.

The stonemason wishes to move large blocks of stone, in order to then cut them into manageable pieces for tombstones. He is considering how to move them most quickly.

In order to move a block, he chooses the initial orientation, and then rolls it in the same direction for the whole journey.

He will only consider blocks that are cuboid in shape and have dimensions that are whole numbers of metres.

(a) Consider a block with dimensions 2 m x 2 m x 6 m. Calculate the minimum possible **time** that it would take to roll the block through 360°. [1]

(b) Consider a block with dimensions 1 m x 4 m x 6 m. Calculate all the different possible **distances** that the block could travel in one 360° revolution, according to the different initial orientations. [1]

(c) If a 24 m^3 block is to travel at least 610 m, what is the smallest possible number of 90° turns that will be needed?
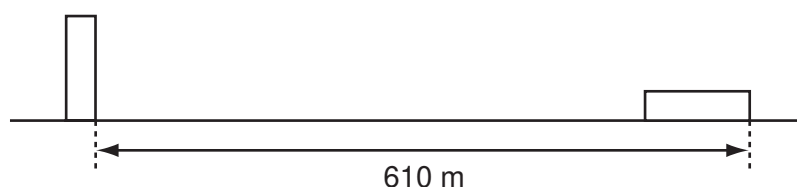
610 m

[3]

(d) What dimensions for a 24 m^3 block will allow for the smallest possible time to move it 610 m? State the time it will take, to the nearest minute. [4]

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He decides that he needs $61\,m^3$ of stone for the next season. He can only move a maximum of $24\,m^3$ at a time. It takes 5 minutes for him to return the $610\,m$ distance without a stone.

(e) Show that it is possible for him to move **exactly** $61\,m^3$ of stone in less than 500 minutes. [4]

(f) He realises that by transporting more than $61\,m^3$ of stone in total, he can reduce the overall amount of time. However, he does not want to move any more than $70\,m^3$ or there will be too much waste.

What set of block sizes should he move to minimise his total time? [2]

4   *Study the information below and answer the questions. Show your working.*

Eightskate is a sport in which two teams of eight compete on roller skates.

An eightskate match consists of 12 heats. In each heat, three skaters from each of the two teams are nominated by the respective team managers to race around a 300 m circuit. The skater who crosses the line last at the end of the second lap drops out. This is repeated at the end of the third, fourth and fifth laps – these retiring skaters being ranked fifth, fourth and third respectively. The remaining pair continues for a sixth lap to determine first and second positions.

An eightskate match between Grinnon Bears and Lowan Sharks is currently in progress and is about to reach its climax. The two team managers will shortly nominate their skaters for Heat 12.

This is the score sheet as it stands at present:

Grinnon Bears v. Lowan Sharks

BEARS	Points Scored												
Heat	1	2	3	4	5	6	7	8	9	10	11	12	Total
T. Black	2				0		1		3		0		
O. Brown		4		4			5			5			
L. Grizzly	1			1		0		4			2		
E. Honey	5		4		5			2					
R. Kodiak			0			3		0		2			
A. Malay		2			1			2			5		
T. Polar			2			4			1				
E. Ursa		3		5			0			4			
Team	8	9	6	10	6	7	6	6	6	11	7		

SHARKS	Points Scored												
Heat	1	2	3	4	5	6	7	8	9	10	11	12	Total
S. Angel		0			2		4		0				
W. Basking	4		3			5			5		3		
I. Hammer		5		3		1		5					
N. Head				2			3			3			
D. Horn		1			4			1		0			
L. Nurse	0		5			2			4		4		
E. Tiger	3			0			2			1			
R. White			1		3			3			1		
Team	7	6	9	5	9	8	9	9	9	4	8		

Points are awarded as follows:

Position	Heats 1 to 11	Heat 12 only
First	5 points	10 points
Second	4 points	7 points
Third	3 points	4 points
Fourth	2 points	2 points
Fifth	1 point	1 point
Sixth	0 points	0 points

When nominating the skaters for any heat, a team manager must be careful to comply with the following rules:

- All skaters must be nominated for a minimum of 4 heats.
- No skater may be nominated for two consecutive heats.
- The same three skaters may not all be nominated together a second time.

(a) After which heat did the Sharks first take the overall lead in the match? [2]

(b) (i) What is the maximum total distance (in metres) that one skater could possibly skate during an eightskate match? [2]

(ii) What is the total distance (in metres) that Edward Ursa has skated so far during this match? [2]

(c) Select a possible line-up of three skaters to compete in Heat 12 for

(i) the Bears, [2]

(ii) the Sharks. [2]

(d) (i) Assuming that no skater drops out, is it possible for any eightskate match to end in a draw? Explain your answer. [2]

(ii) Rupert, a keen Bears fan, fears that his team can only win the match in progress if they secure first position in Heat 12. Is he right? Explain your answer. [3]

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