# MARK SCHEME for the May/June 2012 question paper for the guidance of teachers 

## 9694 THINKING SKILLS

9694/31 Paper 3 (Problem Analysis and Solution), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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1 (a) (i) What is the most he could gain with this strategy?
The highest would come from the maximum increase on the first day: $+3-0.5=\underline{2.5}$.
(ii) What is the most he could lose?

The most he could lose is if the least possible increase happens on the last day: $+1-2=-1$. A loss of 1 penny.
(b) (i) Describe the strategy that Greta selects.

Because it is possible that the gain is only 1 penny, Greta can only risk two nights. [1 mark for noting this limit.]
These must be the last two nights [1 mark] as otherwise she might be left having spent the penny.

If two nights are not suggested, candidates should be awarded 1 mark for recommending waiting until Thursday, or waiting until Wednesday with the possible implication that she would take out a loan on both nights (but not explicitly stated).
(ii) What is the probability that she will make some gain?

If the change is on Wednesday then she gains from all three possible increases, having only put forward half a penny. If Thursday, she only gains from two. So $(3+2)$ of the twelve gives a $\underline{5 / 12}$ chance of a gain.
(iii) What is the most that she can gain?

The most she can gain is 2.5 , (with the maximum increase on Wednesday).
(iv) On Monday, what is the probability that she will have gained the maximum possible by Friday?

There is only one possible way to get the maximum, so $1 / 12$ ( $8.3 \%$ or $8 \%$ permitted).
(c) On Tuesday, what is Harold's chance of having more than 20 pennies on Friday?

2/9 (22.2\% or $22 \%$ permitted) [2 marks].
His 1 penny cost will be exceeded by gains only if the value goes up by 2 or 3 .
There are only three days left so the chance is $2 / 9$.
1 mark if the candidate shows appreciation that in two out of the three scenarios he will make a gain, or one out of three days he could make a gain.
(d) How high would the cost have to be for a loan of 10 pennies before a borrower could not gain by taking a loan on the final night?

If the price is three pennies or more then there is no chance of gaining on the night.

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2 (a) (i) If all 26 letters were in use, for which three pairs of letters would a mistyping of an adjacent key not be detectable?

## A \& S, W \& E, G \& Y.

2 marks for all three correct (and no extras). Deduct one mark if a fourth answer is given.
1 mark for any correct pair (and not more than 3 given).
(ii) Given that the 19 letters specified above are allocated, for which two pairs of countries would a mistyping of an adjacent key not be detectable using the check?

Denmark and Slovakia, Malta and Greece.
Allow $W$ and $E, G$ and $Y$.
(b) (i) What is the largest possible size of a set of letters of the alphabet for which such errors would always be detectable?
23.

1 mark for $26-3$, or $26-6$, or 20, or " 3 missing".
(ii) Give an example of such a set by listing the letters not used.
$S$ or $A$ and $W$ or $E$ and $G$ or $Y$.
(c) A proposed new check only fails to detect shifts along the alphabetical order of multiples of 7 . If the allocation of the letters remains as it currently is, which letter has the most possible undetectable errors when mistyping one of the surrounding keys?

S (two errors, E and Z).
1 mark for any of the letters F, T, E or Z.

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(d) When all 26 letters are used, it is possible to have a system in which any mistyping of an adjacent letter would be detectable, using only three groups. $Q, W$ and $E$ must be in different groups. List all the letters in each of the three groups.

| $\mathbf{Q}$ | $\mathbf{W}$ | $\mathbf{E}$ |
| :---: | :---: | :---: |
| R | T | Y |
| U | I | O |
| P | Z | X |
| C | V | B |
| N | M | A |
| S | D | F |
| G | H | J |
| K | L |  |

2 marks for fully correct.
1 mark if $S, Z$ and $A$ are correctly allocated (with $Q, W$ and $E$ ).

(a) (i) When producing $25 \%$ 'light', the output of 'heavy' is $\mathbf{6 0 0 0}$ barrels per day. What is the total daily profit?

6000 barrels of heavy when $25 \%$ is light means 8000 total.
Thus $\$(12.5-6) \times 8000$ profit $=\$ 52000$.
2 marks for profit.
1 mark for average sale price ( $\$ 12.50$ ) or $\$ 100000$ seen or implied.

## (ii) What proportion of 'light' oil should the refinery choose in order to maximise its profits?

Need to maximise profit for Riebeeck, which is difference between cost and sale price. This is at $50 \%$ light, as it increases below and decreases above.

2 marks for the answer (50\%) clearly stated.
1 mark for either $50 \%$ or $75 \%$ profits calculated correctly (\$56 000 and $\$ 52000$ ).
(b) What is the price for 'light' above which the refinery's profit becomes greatest by producing $\mathbf{7 5 \%}$ 'light' and $\mathbf{2 5 \%}$ 'heavy'?

For $75 \%$ (where we can still use the lower production cost without the extra equipment) $\$ 10+4 \times \$(11-8)=\$ 22$. (Answer sought is the threshold.)

Candidates comparing algebraic statements of the profit at $50 \%, 75 \%$ and $100 \%$ :
Let $x=$ price of light, considering 8000 barrels:
$(6000 x+20000)-(8000 \times 11)>(4000 x+40000)-(8000 \times 8)$
1 mark for one correct algebraic expression for profit
2 marks for an appropriate (in)equality
subtract 1 mark for one or two arithmetic/transcription errors
Stated as a proportion : $(0.75 x+2.5)-11>(0.5 x+5)-8$
Attempts involving trial and improvement. General principle - 1 mark for a trial, and 1 mark for improvement towards threshold.
e.g. at $\$ 24,75 \% / 25 \%$ yields a profit of $\$ 76000$
$50 \% / 50 \%$ yields a profit of $\$ 72000$
Award 1 mark for a solution unambiguously attempting to use the gradient of the $3^{\text {rd }}$ line segment: $(11-8) / 25$
(c) What is the lowest price for 'light' above which the refinery's maximum profit would always be gained by only producing 'light'?

For $100 \%$ price would need to be $\$ 10+4 \times \$(19-11)=\$ 42$.
2 marks for missing $\$ 10$ or using (19-14).
Candidates comparing algebraic statements of the profit at $75 \%$ and $100 \%$ :
Let $x=$ price of light, considering 8000 barrels:
$8000 x-(8000 \times 19)>(6000 x+20000)-(8000 \times 11)$
1 mark for one correct algebraic expression for profit - including ( $6000 x+20000$ ) - (8000 $\times$ 14).

2 marks for an appropriate (in)equality
subtract 1 mark for one or two arithmetic/transcription errors
Stated as a proportion: $x-19>(0.75 x+2.5)-11$
Attempts involving trial and improvement. General principle -1 mark for a trial, and 1 mark for improvement towards threshold.

Award 1 mark for a solution unambiguously attempting to use the gradient between $(100,19)$ and $(75,11)$.

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(d) Determine what the importers' total daily profit would be if they set the price of 'light' at
(i) $\$ 43$ per barrel,
[2]
At $\$ 43$ the Bolandians would only make light, so the importers would only sell heavy at the fixed price of $\$ 10$. Profit is $\$ 4 \times 8000=\$ 32000$.

1 mark for Bolandian profit $=8000 \times(43-19)=\$ 192000$
OR 1 mark for the importers' sales $=8000 \times 10=\$ 80000$
OR 1 mark for 50/50 importers' profit: $(4000 \times(43-17))+(4000+(10-6))=\$ 120000$
OR 1 mark for 75/25 importers' profit: $(2000 \times(43-17))+(6000+(10-6))=\$ 76000$
(ii) $\$ 29$ per barrel,

At $\$ 29$ (between 22 and 42) the Bolandians would make $75 \%$ light. So importers' profit is $6000 \times \$(10-6)+2000 \times \$(29-17)=\$ 48000$.

1 mark for Bolandian profit $=6000 \times(29-11)+(2000 \times(10-11))=\$ 106000$
OR 1 mark for the importers' sales $=6000 \times 10+2000 \times 29=\$ 118000$
OR 1 mark for 50/50 importers' profit: $(4000 \times(29-17))+(4000+(10-6))=\$ 64000$ if equivalent mark NOT awarded in (i).
(iii) $\$ 21$ per barrel.

At $\$ 21$ the Bolandians would make $50 \%$ light. So importers' profit is $4000 \times \$(10-6)+$ $4000 \times \$(21-17)=\$ 32000$.

4 (a) For a game where the SET number is 7, list all the different scorelines that are possible after both players have had one turn.

$$
\text { Answer: } 1-4,3-2,3-5,3-8,3-9
$$

award 2 marks
If 2 marks cannot be awarded, award 1 mark for any of the following:

- Appreciation that 1 and 3 are the only options available to the first player (at least three scores must be given)
- All five correct answers plus $1-7$ and/or 3-7, OR four correct answers but missing one scoreline
- All five correct scorelines, but at least one the wrong way round (e.g. $4-1$ ).
(b) State the four different numbers that the SET number could have been.

Answer: 2, 3, 5 and 6
award 1 mark

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(c) When the SET number is 9, what number should the first player play in order to guarantee a lead of 7 points after both players have had two turns?

Answer: 4 (continuing 1, 7, 3)
award 2 marks
1 mark for any two rounds (involving sub-optimal moves) resulting in a lead of 7 points or more.
(d) Trevor has just changed from 8 to 9. What options are now available to Fiona?

Answer: (She can change from 9 to) 3
(or from 9 to) 5 award 1 mark

Deduct 1 mark for each incorrect number (minimum 0 marks), but award 1 mark in total for one correct number and one incorrect number.
(e) If both players in a game of Digitadd keep scoring the greatest allowable amount every turn, what are the other two possible final scorelines?

Answer: SET 1: $\underline{53-43}$, SET 7: $\underline{49-55}$
3 marks for both scorelines correct.
2 marks for one correct scoreline.
1 mark for evidence of a correct sequence of scores up to the score during the second turn of 9 (in one SET)

SET 1: 7, 3, 9, 8, 6, 9, [scorelines : $7-3,16-11,22-20, \ldots]$
SET 7: 3, 9, 8, 6, 9, 8, [scorelines : $3-9,11-15,20-23, \ldots$ ]
(f) Work out and explain how Fay managed to snatch victory.

Fay changed (from 8) to $\underline{2}(47-42)$,
forcing Simon to change to $\underline{3}(50-42)$,
and finally Fay changed to $\underline{9}(50-51)$.
award 1 mark award 1 mark (dependent on previous one) award 1 mark (dependent on previous two)
(g) When the SET number is 2 , which numbers will not be played by either player during the game?

Answer: 5 and 6.
award 1 mark for each
Deduct 1 mark for each incorrect number (minimum 0 marks), but award 1 mark in total for one correct number and one incorrect number.

