Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**THINKING SKILLS** **9694/32**

Paper 3 Problem Analysis and Solution **May/June 2014**

**1 hour 30 minutes**

Additional Materials: Electronic Calculator

*2946340790*

**READ THESE INSTRUCTIONS FIRST**

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Calculators should be used where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages, **1** blank page and **1** insert.



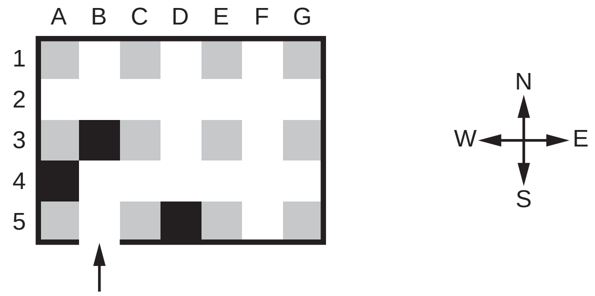

CAMBRIDGE International Examinations

**[Turn over**

1 *Study the information below and answer the questions. Show your working.*

The following diagram shows the design for a toy. It represents a solid box with a single entry point (shown with an arrow) through which a ball is inserted into the maze.

Inside the box there are five straight, intersecting passageways, represented by white areas in the diagram. To create the maze a number of bungs, represented as black squares in the diagram, are placed inside the passageways, ensuring that no part of the maze is blocked off (i.e. no white space is unreachable by the ball). In the diagram, bungs have been placed at the squares labelled A4, B3 and D5.



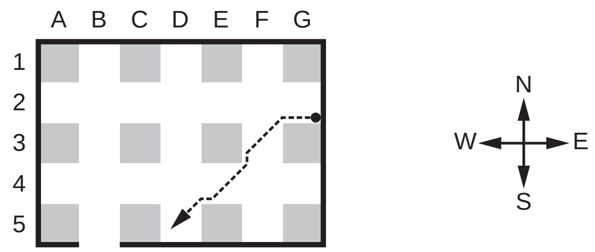

**(a)** When creating a new maze, with no bungs currently in position, in how many different places can the first bung be put without blocking off any section of the maze space? [2]

**(b)** List all the pairs of places where just two bungs would block off a section of the maze, even though neither bung on its own would. [2]

**(c)** A white square is called a 'pocket' if it is open on only one side (e.g. B1 in the diagram above). What is the greatest and least number of pockets that a maze could have, if it contains three bungs, and no areas are blocked off? [2]

Once the bungs have been placed, the ball is inserted. The box is shaken around, and the player is given the task of trying to get the ball out, by tilting the box and thus making the ball roll in different directions. The player must wait for the ball to stop before changing the direction of tilt.

The maze can be tilted in 8 directions, which can be referred to using the standard compass directions. When the maze is tilted in one of the four principal directions (N, E, S, W), the ball moves in that direction until it reaches an obstruction, where it stops. If it is tilted in one of the other four directions (NE, SE, SW, NW), the ball moves diagonally where possible, but if not it moves at 45° to the direction of tilt. An example of the effect of a south–west tilt is shown in the diagram below.




**(d)** Consider the maze represented at the beginning of the question, with bungs at A4, B3 and D5. State the starting square from which the ball cannot escape with fewer than three tilts. List three directions, in the order that they occur, that will allow the ball to escape. [2]

**(e)** State where **two** bungs should be placed, and where the ball should start, if it is to require a route involving four tilts in order to get out. [2]

**2** *Study the information below and answer the questions. Show your working.*

A door has a lock which is operated by entering a four-letter code into an electronic keypad. There are 4 buttons on the keypad, labelled A, B, C and D. The program in the keypad operates by moving between five different states.

Every time that a button on the keypad is pressed the program either moves to a different state or remains in its current state. When it moves into state 5 the door is released and the program moves back to state 1.

The table below shows the next state for the program when different inputs are made while it is in states 1 to 4.

| | | *Input* | | | |
|---|---|---|---|---|---|
| | | A | B | C | D |
| *State* | 1 | 1 | 1 | 2 | 1 |
| | 2 | 3 | 1 | 2 | 1 |
| | 3 | 1 | 1 | 4 | 1 |
| | 4 | 3 | 1 | 2 | 5 |

The correct first character of the code is C (which moves the program from state 1 to state 2).

Mark attempted to unlock the door by entering the sequence C B C A. When the C button was pressed for the first time the program moved to state 2.

**(a)** Show that Mark has left the program in state 3. Write down which state the program was in after each time Mark pressed a button. [2]

**(b)** What two further button presses would unlock the door? [1]

If the program is in state 2, then pressing either B or D will return the program to state 1, as neither is the correct second character of the code. C is also incorrect, but returns the program to state 2 because C is the correct first character of the code.

**(c)** Explain why the program is designed so that it will always be in state 1 after B is pressed. [1]

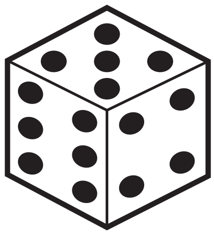
**(d)** Explain why the program is designed so that pressing A while the program is in state 4 will return the program to state 3, rather than state 1. [2]

Another lock is programmed to open only when the code AABDAC is entered. The program for this lock uses seven states. When it moves into state 7 the door unlocks and the program immediately moves back to state 1.

**(e)** Draw a table to show the next state for the program when different inputs are made while it is in states 1 to 6. [4]

**3** *Study the information below and answer the questions. Show your working.*

Tossing a coin is not a good way to make a fair random selection of one of three people, but a fair choice can be made by rolling a standard die.



**(a)** Suggest a simple way to choose fairly between three people, using a single roll of a die. [1]

Alexander noticed that a single roll of a die is sufficient to select fairly from 12 possible options. Since there are 12 edges, he could do this by looking at the vertical edge which is nearest to him. Each of the 12 options could be associated with one of the 12 edges. Charlotte was worried that sometimes the die would land with a face 'square on'. They decided that in such cases they would always take the edge to the right, as this would not introduce any bias.

Zoe pointed out that she could do the same much more easily by rolling the die, then considering both the number on top and whether the vertical edge nearest her was on the right or left of centre. Each of the 12 options could be associated with a number and a direction (1L, 1R, 2L, 2R etc.). Alexander then worked out how to choose one from 24 options fairly using a single roll.

**(b)** Describe a method he could use. (Do not list all the cases.) [2]

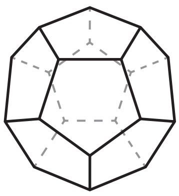
Charlotte suggested subtracting the number on the left visible vertical face from the one on the right. In the case shown above that would give 4 – 6 = –2. (Note that opposite faces of a die add to 7.)

**(c)** **(i)** How many different results are possible? [2]

This method does not give the same chance of each result.

**(ii)** How many times more likely is a difference of 2 than a difference of 3? [2]

Donald obtained a regular twelve-faced solid (dodecahedron) with faces numbered 1 to 12, and on which opposite faces always add to the same sum.



**(d)** What is the sum of opposite faces? [1]

Donald could use the 12 faces to select one of 2 options, or one of 3, 4, 6, or 12 options.

John was much more interested in using a dodecahedron to choose one of 5 players to start a game.

**(e)** How can John achieve this in an unbiased way using a single roll? [3]

**(f)** **(i)** Suggest a way for Alexander to get 60 equally likely options from one roll of a dodecahedron. [1]

**(ii)** Zoe finds a way to get 120. How can she do this? [1]

**(g)** Charlotte says that by rolling the standard die and the dodecahedron once each, she can fairly choose between 45 options. Suggest a way that she could achieve this. [2]

**4** *Study the information below and answer the questions. Show your working.*

Trypples is a game for two players, played over a number of rounds. It involves the placing of tiles onto a 3 x 3 grid of squares.

There are 12 tiles, as follows:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 |

Before the game, the toss of a coin decides who will be player X and who will be player Y.

In each round, the players take turns to remove one tile, at random, from a bag and place it onto one of the empty squares on the grid, until the grid is complete. The player placing the first tile alternates from round to round, with player X going first in round one.

The object of the game is to create lines of three different numbers. Each such line scores the sum of the three numbers. Lines in which a number appears twice score zero. Throughout the whole game, player X scores points for all (horizontal) rows and player Y scores points for all (vertical) columns. For example, in the completed grid shown below, player X scores 17 points (1 + 2 + 6 and 1 + 5 + 2) and player Y scores 11 points (6 + 3 + 2).

| | | |
|---|---|---|
| 1 | 2 | 6 |
| 3 | 5 | 3 |
| 1 | 5 | 2 |

Points scored are added to the players' totals after a grid has been completed.

The game is won by the player who is first to lead by at least 10 points, having scored 100 points or more.

Xavier and Yvette are playing a game of Trypples. Appropriately, Xavier is player X and Yvette is player Y.

This was the grid at the end of the first round.

| | | |
|---|---|---|
| 5 | 1 | 6 |
| 4 | 3 | 3 |
| 5 | 4 | 2 |

**(a)** **(i)** How many points did Xavier score in the first round? [1]

**(ii)** How many points did Yvette score in the first round? [1]

In the second round, Yvette scored the maximum possible score of 38 points, whilst Xavier scored zero.

**(b)** Draw a completed grid that would produce this result. [2]

**(c)** Explain why it is **not** possible for both players to score zero in a round of Trypples. [2]

In the third round Xavier placed the first tile, which was a 3, in the centre square. The players eventually drew this round 10–10, with only the middle row and the middle column scoring points.

**(d)** Which three tiles were left in the bag at the end of the third round? Explain how you can be sure it was these three tiles. [3]

This was the grid during the fourth round, after both players had placed two tiles:

| | | |
|---|---|---|
| | 1 | |
| 6 | 4 | 4 |
| | | |

Xavier won the fourth round 21–7.

**(e)** Draw the completed fourth round grid. [3]

After the sixth round, Yvette was leading with 83 points to Xavier's 79. Currently, in the seventh round, the grid is as follows:

| | | |
|---|---|---|
| 6 | 1 | |
| | 3 | 1 |
| 3 | | |

Yvette has just taken the second 6 from the bag, and has realised that she will win the game at the end of this round if she places the 6 in the top right square.

**(f)** Explain fully how Yvette can be sure that she will win with this move. [3]

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