Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

THINKING SKILLS **9694/31**

Paper 3 Problem Analysis and Solution **October/November 2015**

2 hours

Additional Materials: Electronic Calculator

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Show your working. Marks may be awarded for correct steps towards a solution, even if the final answer is not correct. Marks may be lost if working needed to support an answer is not shown.

Calculators should be used where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 There was only one ice-cream left, and Mark, John and Luke each wanted it. Their first idea was for each of them to toss a coin once, and if only one person had a head or only one person had a tail then that person would get the ice-cream.

(a) What is the chance that this approach would **not** decide who gets the ice-cream? [2]

Mark then suggested that the other two should each select a number: 1, 2, or 3. He would give the ice-cream to the person who selected the higher number if they were different, but if they were the same he would eat it himself.

(b) (i) If John and Luke had each selected a number at random, what would Mark's chance of getting the ice-cream have been? [1]

(ii) What would Luke's chance of winning have been if he had selected 1? [1]

John knew that Luke would avoid 1 and toss a coin to select either 2 or 3.

(c) (i) Knowing what Luke would do, what was the best strategy for John to maximise his chance of getting the ice-cream, and what was his chance of success? [2]

(ii) What was Luke's chance of getting the ice-cream if John used this strategy? [1]

(iii) If Luke and John both avoid 1 and each, independently, toss a coin to select 2 or 3, what is the chance that John will get the ice-cream? [1]

When they found that there was often going to be only one ice-cream left, Luke and John noticed that they could play Mark's game and agree a strategy so that Mark would never get the ice-cream. Luke and John would each get it half of the time, without having to communicate after the strategy was agreed.

(d) Describe a simple strategy for them to use. [1]

Working together, Luke and John could have ensured that John always got the ice-cream, or that Luke always did (or even that Mark always did). However, we can assume that they each want as much as they can get.

(e) Is there a strategy that Luke can use so he can expect to get the ice-cream more than half the time, whatever John does? Explain your reason. [1]

2 Each day, sliced bread loaves are delivered to a boarding house for the residents to make toast. There are 20 slices per loaf. Any bread from the previous day is used first.

Fred assumed that 88 slices would be used each day. He intended to place an order for the first three weeks for either 4 or 5 loaves each day. If Fred's assumption was correct, then there would always be enough, but never a whole loaf left at the end of a day.

For the first 21 days the order would have been:

Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su
5	4	5	4	4	5	4	5	4	4	5	4	5	4	4	5	4	5	4	4	5

(a) (i) How many slices of bread would be left at the end of the third day? [1]

(ii) When was the first day when there would have been no bread left at the end of the day? [1]

(iii) How many slices would be left at the end of the first three weeks? [1]

The bakery told Fred that, because the actual daily consumption was likely to vary, his order could result in there not being enough on some days. They proposed that each delivery should be based on the number of slices left over at the end of the previous day. Fred agreed with their proposal. The numbers of loaves delivered and slices consumed each day are shown in the table below.

Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su
5	4	5	4	4	5	4	4	4	5	4	4	5	4	4	5	4	5	4	5	4
86	88	84	91	81	84	86	86	88	85	84	87	87	89	91	86	92	88	88	89	87

(b) (i) Did this approach lead to more or less bread being bought over the first three weeks, or was it the same? [1]

(ii) By how many did the total number of slices consumed over the first three weeks differ from Fred's assumption? [1]

(c) Would Fred's original plan have resulted in a shortage, and, if so, on which day? [1]

(d) (i) What was the minimum number of slices left over that resulted in the bakery delivering 4 loaves? [2]

(ii) Does this guarantee there will be enough every day if a similar usage continues? Explain your answer. [2]

3 A game for four players involves buying tokens. The aim of the game is to buy more tokens than any other player. In any round in the game there is a number of tokens for sale and this determines the price that has to be paid for each.

- If there is only one token for sale it costs \$99.
- Every extra token for sale reduces the price by \$1 until the price has reduced to \$50.
- From this point the price reduces by \$0.50 for every extra token for sale.

In the first round of a game, there are always 125 tokens for sale and each player has \$1000.

(a) How much does each token cost in the first round of the game? [1]

At the start of each round all of the players indicate how many tokens they want to buy at that round's price and they pay for that number. If the total requested is more than the number for sale then a maximum allocation per person is determined for that round. The tokens are then distributed as follows:

- Any player who requested fewer than the maximum allocation gets the full number requested.
- Any player who requested the maximum allocation or more gets the maximum allocation.

The maximum allocation per person is chosen to be the largest value possible. This process may mean that some tokens are not sold even though more than the number for sale were requested. There is no refund for the tokens that were paid for, but were not received.

For example, in one particular round of a game only 20 tokens were available. Each token therefore cost \$80. John paid \$400 for 5 tokens, Bill paid \$720 for 9 tokens, Sue paid \$800 for 10 tokens and Terry did not request any tokens. Because there were not enough tokens to give every player the number that they requested, John received 5 tokens and Bill and Sue each received 7 tokens. 1 token remained unsold.

(b) If all four players in a game each requested 40 tokens in the **first round**, what would the maximum allocation per person be? [1]

(c) If the four players in a game requested 20, 26, 50 and 80 tokens in the **first round**, what would be the numbers of tokens that the four players receive? [2]

(d) In the first round of another game, Joseph requested 80 tokens but only received 40. Give an example of the numbers of tokens that could have been requested by the other three players if 2 tokens remained unsold. [2]

At the end of each round 10 more tokens are added to those not sold in the previous round. The game ends when no player requests any tokens or there are at least 150 tokens for sale. The player with the most tokens wins the game.

(e) Eliza knows that each of her opponents will request just one token in any round where the price is \$10 or more, and request 50 tokens if the price is less than \$10. Show that Eliza can receive a total of 99 tokens by the end of the second round. [2]

(f) At the start of round 5 in another game the price of tokens was \$5.50 less than it had been in round 4 of the game. Give an example of the number of tokens available at the start of round 4 and the total number requested in that round that could lead to this. [3]

(g) Near the end of one game Jill is the only player with any money left. She has $50 and at the start of the current turn there are 130 tokens available. What is the largest number of extra tokens that she can buy before the game ends? [4]

4 These are the scores from the matches played yesterday in the Corvenian Football League, and the league table as it stands today.

Crows	2	Jackdaws	3
Jays	1	Ravens	0
Nutcrackers	0	Magpies	2
Rooks	2	Choughs	2

	Played	*Won*	*Drawn*	*Lost*	*First goal*	*Points*
Ravens	13	4	8	1	6	36
Jays	13	6	3	4	8	35
Choughs	13	5	4	4	7	35
Nutcrackers	13	5	2	6	7	34
Jackdaws	13	5	3	5	5	32
Crows	13	4	4	5	4	29
Magpies	13	5	1	7	6	28
Rooks	13	4	3	6	5	27

The teams all play each other twice every season, and points are awarded for each match, as follows:

4 points for a win by a margin of two goals or more
3 points for a win by a margin of one goal
2 points each for a draw
1 point for a loss by a margin of one goal
0 points for a loss by a margin of two goals or more

In addition, 1 point is awarded to the team that scores the first goal of the match.

Where two teams have the same number of points, the team with the greater number of 'first goal' points is placed higher in the league table.

Next week's matches, the last of the season, are:

Choughs	v	Jays
Magpies	v	Ravens
Jackdaws	v	Rooks
Nutcrackers	v	Crows

(a) How many points did the Jays gain from their match yesterday? [1]

(b) Which team had lost exactly half of its previous matches this season, before yesterday? [1]

(c) How many matches have the Ravens won this season by two goals or more? Justify your answer. [2]

(d) Of all the matches played so far this season, how many have been 0 – 0 draws? [2]

When the Choughs played the Jackdaws three weeks ago, 7 goals were scored during the match, with the Choughs gaining 3 points and the Jackdaws 2 points.

(e) Give the final score of this match, and explain how this score resulted in the Choughs gaining 3 points and the Jackdaws 2 points. [2]

(f) Any one of the top four teams could still win the league. Explain why the Jackdaws **cannot** win the league. [2]

A record number of draws for this league has helped the Ravens to stay at the top, despite only winning four of their matches.

(g) Give a set of scores for next week's matches that would result in the Ravens winning the league with fewer matches won than any of the other teams, regardless of which teams score first. [3]

(h) There has been a proposal to allow two more teams to join the league from next season. If this proposal is accepted, how many more matches will be played altogether during a season than at present? [2]

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