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THINKING SKILLS 9694
GCE A/AS Level FOR EXAMINATION IN 2008
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## THINKING SKILLS

## GCE Advanced Subsidiary Level and GCE Advanced Level 9694

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## 1. Introduction

The teaching of Thinking Skills is becoming more widespread in the modern school curriculum. It involves the learning of a specific set of intellectual skills independent of subject content. This reflects the need to encourage students to develop more mature and sophisticated ways of thinking. By taking a course in Thinking Skills and applying these skills to their wider academic learning, it is hoped that students will approach their other subjects with an improved ability to understand, analyse and resolve problems.

Candidates will find Thinking Skills of great benefit in preparation for higher education and a wide range of careers, including the fields of law, scientific research, social science, journalism, medicine, business, accounting and engineering. Universities are increasingly looking for students who can demonstrate a high level of objective and rigorous thinking.

As a curriculum subject, Thinking Skills offers students an excellent opportunity to express themselves in a free and open fashion. Lively debate, critical and investigative thinking are to be encouraged in the lessons, coupled with informed and disciplined reasoning.

The Thinking Skills syllabus has two aspects: Problem Solving and Critical Thinking. Each of these consists of a set of sub-skills. These are explained in more detail below.

## Problem Solving

The problem solving component is designed to assess a student's ability to analyse numerical and graphical information in the context of real life situations and apply appropriate numerical techniques in order to find new information or derive solutions.

Problem solving draws on a range of skills such as data handling, reading, modelling, logic and reasoning. In the CIE syllabus, the problem solving abilities of candidates are assessed at the level of these various sub-skills, which are the building blocks for successful application to wider and more complex problems.

For the examination students need to be able to apply simple mathematics to new situations in order to demonstrate an ability to manipulate numerical and graphical data. They will need to be able to extract and use relevant data and find methods of using information in order to come to conclusions. Students are also required to recognise how the same data may be presented in different forms. They are expected to be able to think critically about information, evaluate possible reasons for unexpected variations and be able to use information for informed decision-making.

The Thinking Skills AS papers assume that students are familiar with basic techniques of mathematical manipulation to junior school level. The Thinking Skills A Level papers assume that students are familiar with basic techniques of mathematical manipulation to IGCSE/O level standard. Thinking Skills is not designed as a test of students' mathematical abilities. Rather, the Problem Solving element of this subject is about using logical methods of handling numerical, graphical and pictorial data. Problem solving skills are not only desirable but essential to lawyers, sociologists, geographers, historians and those in many other professions. They have to understand and use numerical information, to analyse it and to draw conclusions from it. For example, a lawyer may need to know about and understand the probabilities of a DNA test being conclusive. Sociologists frequently employ statistics and demographic data. The incorporation of Problem Solving is a major strength of this syllabus.

## Critical Thinking

Central to Critical Thinking is the notion of argument. From the start students should learn to recognise when someone is engaged in reasoned argument, as distinct from quarrelling, disputing, reporting or explaining. Different examples of reasoning and argument need to be explored in order to understand their common characteristics, and most importantly the use of reasons (or premises) to support conclusions.

Candidates should acquire a basic language of reasoning: everyday words such as 'therefore', 'because', 'if...then', which are used in arguments; and semi-technical terms such as 'conclusion', 'assumption', 'flaw', 'sufficient', which are used to talk about argument.

The main activities which comprise Critical Thinking are analysis, evaluation and construction of argument. By analysing arguments students learn to identify the key elements of a reasoned case, and to understand how they function. Evaluation involves making informed judgements about the soundness, strength or weakness of a piece of reasoning. This frequently includes assessing the impact of responses to an argument: challenges, supporting evidence, counter-examples, etc. In addition candidates are required to engage in their own reasoning, based on given stimulus material.

## 2. Aims

The aims of the Thinking Skills syllabus are:

- To provide students with a specific and transferable set of skills for solving problems, critical thinking and reasoning.
- To encourage students to apply these skills to realistic scenarios.
- To develop students' abilities to understand and engage confidently in argument and reasoning.

The 'specific skills' referred to in these aims are set out in the following table.

|  | Critical Thinking | Problem Solving |
| :---: | :---: | :---: |
| Core | - Recognizing reasoned arguments (skill CT1) <br> - Identifying conclusions (CT2) <br> - Drawing conclusions (CT3) <br> - Recognizing implicit assumptions (CT4) <br> - Assessing the impact of further evidence (CT5) <br> - Recognizing flaws in reasoning (CT6) <br> - Selecting plausible explanations (CT7) <br> - Recognizing the logical functions of key elements of an argument (CT8) <br> - Understanding and clarifying key terms and expressions (CT9) <br> - Engaging in inference and deduction (CT10) <br> - Analyzing and evaluating evidence and argument (CT11) <br> - Constructing argument (CT12) | - Data selection and processing (skill PS1) <br> - Finding methods and procedures (PS2) <br> - Identifying relationships and hypotheses (PS3) <br> - Analyzing and modelling (PS4) <br> [Mathematics ability assumed: Junior School] |
| Extension | - Constructing a reasoned case, including selecting and synthesizing information (CT13) | - Developing a model (PS5) <br> - Carry out an investigation (PS6) <br> - Analyze complex data and draw conclusions (PS7) <br> [Mathematics ability assumed: IGCSE/O Level] |

Advanced Subsidiary Syllabus = Core
Advanced Level Syllabus = Core + Extension

## 3. Assessment

The CIE AS in Thinking Skills is awarded on the successful completion of Papers 1 and 2.
The CIE A Level in Thinking Skills is awarded on the successful completion of Papers 1, 2, 3 and 4.

Paper 1: Problem Solving and Critical Thinking is assessed through 40 multiple choice questions, each made up of a stimulus, a question and five options - only one of which is the correct answer. Candidates must choose the option that gives the correct answer.
Paper 1 tests skills CT1 - CT9 and PS1 - PS4.

Paper 2: Critical Reasoning is assessed by means of four questions requiring written answers. Each of questions 1 to 3 is based on a short passage and/or information in graphical or numerical form. The passage is followed by 4 or 5 sub-questions, each requiring a short written answer. Question 4 is based on a passage of about 350 words and requires a longer structured written answer.
Question 1 requires the candidate to evaluate evidence.
Question 2 requires the candidate to understand and evaluate an argument.
Question 3 requires the candidate to engage in reasoning in a scientific context.
Question 4 requires the candidate to evaluate and present an argument.
Paper 2 focuses on skills CT10 - CT12 (but may also test skills CT1 - CT9 and PS1 - PS4).

Paper 3: Problem Solving and Critical Thinking (Advanced) is assessed through 25 questions. These are multiple-choice (requiring candidates to select one or more options as being correct). Each question is based on information in text, graphical and/or numerical form.
Paper 3 tests skills CT1 - CT9 and PS1 - PS4.

Paper 4: Applied Reasoning is assessed by 3 questions.
Question 1 is a problem solving question requiring candidates to develop a model or carry out an investigation.
Question 2 is a problem solving question requiring candidates to analyse information and draw conclusions.
In both questions 1 and 2 candidates are given information in written, graphical or numerical form, followed by 3 or 4 sub-questions each requiring a numerical or short written answer.
Question 3 is a critical thinking question requiring candidates to select from given information, opinion and/or argument and use this to construct a reasoned case. The source material for question 3 will consist of 3 or more documents.
Paper 4 focuses on skills CT13 and PS5 - PS7 (but may also test skills CT1 - CT9 and PS1 - PS4).

## 4. Scheme of Assessment

## ADVANCED SUBSIDIARY QUALIFICATION

| Paper | Type | Duration | Number of <br> Questions | Maximum <br> Mark | Weight (\% of <br> total marks for <br> the syllabus) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Multiple <br> Choice <br> Questions <br> (MCQ) | 2 h | 40 | 40 | 50 |
| 2 | Short answers <br> and essay | 2 h | 4 | 50 | 50 |

Paper 1 is called 'Problem Solving and Critical Thinking' and the 40 questions each carry equal marks.
Paper 2 is called ‘Critical Reasoning’ and the 4 questions each carry similar marks.

## ADVANCED LEVEL QUALIFICATION

| Paper | Type | Duration | Number of <br> Questions | Maximum <br> Mark | Weight (\% of <br> total marks for <br> the syllabus) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | Multiple <br> Choice <br> Questions | 2 h | 40 | 40 | 25 |
| 2 | Short answers <br> and essay | 2 h | 4 | 50 | 25 |
| 3 | Mulitiple <br> Choice <br> Questions | 1.5 h | 25 | 50 | 25 |
| 4 | Short answer <br> and essay | 1.5 h | 3 | 50 | 25 |

Paper 3 is called 'Problem Solving and Critical Thinking (Advanced)' and the 25 questions each carry equal marks.

Paper 4 is called 'Applied Reasoning'. Questions 1 and 2 carry 10 marks each. Question 3 will be marked out of 30 .

## 5. Paper 1: Problem Solving and Critical Thinking

### 5.1 Problem Solving

## SKILLS ‘CONTENT’

The questions test the candidate's abilities to use the following skills.

## Select and process data:

- Extract relevant data
- Process data

Find methods and procedures:

- Find procedures for solving problems
- Search for solutions

Identify relationships and hypotheses:

- Identify similar data
- Suggest hypotheses for variations
- Engage in spatial reasoning

Engage in analysis and modelling:

- Identify necessary and sufficient data
- Choose appropriate models
- Make choices and decisions


## TYPES \& EXAMPLES OF QUESTIONS

The questions are in multiple choice format with a short stimulus passage. The candidates have to select the most appropriate response from five options A to E .

Information may be presented in five categories: tabular, graphical, verbal, pictorial, diagrammatic.
The stimulus may include information in more than one category. The stimulus material will be based on a realistic scenario. In general the answer should be derivable from the stimulus, rather than candidates having to work backwards from the list of possible answers. However, there will be some questions where the options in effect form part of the stimulus. Candidates are not expected to have any subject specific knowledge, for example knowledge of physics or statistics, beyond the basic mathematics required.

Each of the 10 sub-skill categories for Problem Solving is explained below. An example of a question that tests the sub-skill is also provided.

## 1. Data Selection and Processing

## a). Data Extraction

Candidates will be presented with information in verbal, tabular or graphical form. Most will be quantitative but some qualitative elements may be included (e.g. colour or shape). They will also be presented with criteria for choosing one item or a range of items from the data presented. The skill will be to select the correct item or range of items from a set which includes an amount of redundant data. This category may also include a small amount of simple processing, e.g. to find a journey time from a timetable giving departure and arrival criteria: the correct departure and arrival time should be found from the timetable and the journey time found by subtraction.

## Sample question

The cost of sending letters from the United Kingdom to continental Europe is shown below.

| Not over | $£ \mathrm{p}$ | Not over | $£ \mathrm{p}$ | Not over | $£ \mathrm{p}$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 20 g | 0.22 | 250 g | $1 \cdot 06$ | 500 g | 2.02 |
| 60 g | 0.37 | 300 g | $1 \cdot 25$ | 750 g | $2 \cdot 77$ |
| 100 g | 0.53 | 350 g | $1 \cdot 44$ | 1000 g | 3.52 |
| 150 g | 0.70 | 400 g | 1.64 | 1250 g | 4.07 |
| 200 g | 0.88 | 450 g | 1.83 | 1500 g | 4.62 |

A firm in London wishes to send two letters to separate clients in continental Europe. The letters weigh 75 g and 215 g .

What is the total cost of sending the two letters?
A $£ 1.25$
B $£ 1.41$
C $£ 1.43$
D $£ 1.44$
E $£ 1.59$

## Key E

## Stimulus Type Tabular

## Justification

The 75 g letter will cost 53 p to post (over 60 g but under 100 g ) and the 215 g letter will cost $£ 1.06$ p (over 200 g but under 250 g ). The total cost is $53 \mathrm{p}+£ 1.06 \mathrm{p}=£ 1.59$ p.
Candidates are expected to select the two correct values from the table, given the weights of the parcels, and add these together. The primary skill is extraction with a small amount of processing.

## b). Data Processing

Candidates will be presented with information in verbal, tabular or graphical form. They will also be presented with a question which requires use of this data. One or more items from the data set should be used in the correct manner to give an answer. The method of using the data will be clear and not require any elucidation of the correct procedure to use. The skill is in choosing and using the data correctly. This category of question may rely on an element of data extraction as well as processing but this will not be the main emphasis of the question.

## Sample question

We had 76 people wanting coffee at a conference. The caterers provided enough coffee for each of 80 people to have an 8 fluid ounce cup three-quarters full. We carefully filled each cup to exactly three-quarters full as we handed them out, but failed to notice that they had given us 10 fluid ounce cups.

How many people went without?
A 0
B 4
C 8
D $\quad 12$
E $\quad 16$

## Key D

## Stimulus Type Verbal

## Justification

The amount of coffee provided by the caterers was 80 (people) x 8 (fluid ounce cups) $x 3 / 4$ (full) $=480 \mathrm{fl}$ oz. The amount of coffee in one of the larger cups is 10 (fluid ounces) $x 3 / 4$ (full) $=7.5 \mathrm{fl} \mathrm{oz}$.

The number of people that can be catered for is 480 (the amount of coffee provided) / 7.5 (in each cup) $=64$.

There are 76 people so $76-64=12$ people will go without.
The candidate must use the data correctly (there is no redundant data so the only extraction skill is to use the correct numbers at the correct time). The skill is processing. The element of finding a method is minimal, as the method of solution is straightforward.

## 2. Finding Methods and Procedures

## a). Finding a Procedure

Candidates will be presented with information in verbal, tabular, graphical, pictorial or diagrammatic form. They will also be presented with a problem to solve (this will in most cases have a numerical solution but it may also be in other forms e.g. a spatial manipulation). The method of solution of this problem will not be immediately obvious and the candidate will have to determine a suitable method of using the data to come up with the required answer. The skill required is to determine how to use the data in a suitable manner to solve the problem and then to produce a solution.

## Sample question

Each of two identical cars can carry enough fuel to travel 100 miles only. To make a longer journey over a deserted area, they set out together and then at some stage the first car transfers fuel to the other and returns home. The second car travels on.

What, approximately, is the furthest distance from home that the second car can travel?

A $\quad 125$ miles
B $\quad 133$ miles
C $\quad 150$ miles
D $\quad 167$ miles
E $\quad 175$ miles


#### Abstract

Key B Stimulus Type Verbal

\section*{Justification}

A method must be developed to solve this problem - it cannot be done just using extraction or processing. One way is by trial and error. For example, if the two cars travel 50 miles together, it would be possible to transfer 50 miles worth of fuel from one to another but then the car donating the fuel would be empty so this is clearly too far. Similarly, at 25 miles only 25 miles worth could be transferred so the donor car would have 25 miles left when it reaches home.

It rapidly becomes clear that the donor car must divide its fuel into 3: $1 / 3$ each for the outward journey, to transfer to the other car and to return. At 33 miles the car which is continuing is completely refilled and would have enough for another 100 miles, making 133 miles in total.


## b). Searching

Candidates will be presented with information in verbal, tabular, graphical, pictorial or diagrammatic form. They will then have to solve a problem by searching for a solution. Searching can be in two forms. It may be necessary to search the data given to find the correct pieces of information to use. In general it will also be necessary to consider several possible scenarios which may solve the problem, in order to determine which, for example, is the optimum according to a given criterion. Identifying a suitable method of searching and carrying this out correctly are the required skills.

## Sample question

A private mail delivery company makes the following charges for delivering letters and packages:

| Weight up to 60 g | 25 pence |
| :--- | ---: |
| Each extra 10 g or part thereof | 5 pence |

A woman wishes to use this company to send a manuscript either as a single package of weight 138 g or two or more packages with a total weight of 138 g .

What is the lowest cost of postage with this company?
A 59 pence
B $\quad 60$ pence
C 64 pence
D $\quad 65$ pence
E $\quad 75$ pence

## Key B

## Stimulus Type Verbal/Tabular

Justification
The skill is in performing a search of the options on splitting up the package to find how many pieces, and of what weights, is most effective.

As a single package it would cost 25 p for the first 60 g and $8 \times 5 \mathrm{p}$ for the remaining 78 g , or 65p in total.
If sent as two packages, it is most economical to have both at 60 g or more, as the first 60 g is pro-rata cheaper than the remaining weight. Thus $60 \mathrm{~g}+78 \mathrm{~g}$ is as effective as any other split: this costs $25 p+(25 p+2 \times 5 p)=60 p$.
If sent as three packages, once again it is best to keep as many as possible at 60 g or more. They would then divide as $60 g+60 g+18 g$ at $3 \times 25 p=75 p$.
The two package option is the best at 60p.

## 3. Identifying Relationships and Hypotheses

a). Identifying Similarity

Candidates will be asked to match information in one form to the same information in another form (e.g. tabular to graphical), or in a different representation of the same form (e.g. matching a timedistance graph with the same data presented as a time-velocity graph). The information presented and derived can be verbal, tabular, graphical, pictorial or diagrammatic. The skill required from the candidate is to be able to recognise the correct data item or data set from several others which are presented.

## Sample question

The table shows the age distribution of the inhabitants of a small town:

| Age group | $0-15$ | $16-25$ | $26-45$ | $46-64$ | $65+$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of <br> inhabitants | 2997 | 4432 | 3761 | 5980 | 7440 |

Which of the following charts, if suitably labelled, could represent the data above?


## Key A

## Stimulus Type Tabular

## Justification

The second bar should be approximately 1.5 x the height of the first, the third should be in between the first and second, the fourth approximately twice the height of the first and the fifth the largest.
Only chart A shows this.

## b). Suggesting Hypotheses

Candidates are presented with information, usually in graphical or tabular form. This will typically illustrate how one variable changes as a function of another or with time. They must select from a list of choices a statement or an additional piece of data which would explain the nature of variation in the data. The skill is to make a reasoned choice of a possible cause of variation.

## Sample question

Tina is a part-time teacher. She teaches her lessons during the day and, when she has finished her last lesson, she leaves the school and goes to the bus stop to get a bus home. All lessons start on the hour, or at quarter past, half past or quarter to the hour and each lasts 45 minutes. It takes her 15 minutes from the end of a lesson to reach the bus stop. She doesn't know what the bus timetable is, but she notices that she always has to wait either 5 minutes or 20 minutes for a bus.

Which of the following explains the times she has to wait?
A The buses run every 15 minutes at $5,20,35$ and 50 minutes past the hour
B The buses run every half hour at 5 and 35 minutes past the hour
C The buses run every 20 minutes at 0,20 and 40 minutes past the hour
D In the morning the buses run every hour at 5 minutes past the hour; in the afternoon they run every hour at 20 minutes past the hour
E The buses run exactly 35 minutes apart

## Key B

Stimulus Type Verbal
Justification
She always arrives at the bus stop on an exact quarter hour - on the hour or quarter past, half past or quarter to the hour (the lessons start on a quarter hour, take $3 / 4$ hour and she takes 15 min to get to the bus stop).

If she arrives at the bus stop at the hour or half past, she waits five minutes. If she arrives at quarter past or quarter to, she waits 20 minutes. The buses are at 5 and 35 minutes past the hour. B explains this.

The skill is in matching the verbal stimulus to one of the verbal explanations.

## c). Spatial Reasoning

Candidates are presented with information, usually in pictorial form (although other forms such as a verbal description may be used). They are then asked to identify the result of a spatial operation such a deformation, reflection or rotation or, for example, to identify a view from a different direction. The skill is in being able to visualise shapes and views, and how they may change, in two or three dimensions.

## Sample question

The diagram shows a factory. There is a square chimney in the south-eastern corner of the factory. There is an entrance in the western end which can be seen in the diagram. There are two windows in the southern wall. There is also a door in the northern wall of the factory and a window in the eastern wall of the factory, which are hidden in the diagram.


Which of these is a possible view of the eastern end of the factory?



D

E


## Key D

Stimulus Type Pictorial and Verbal

## Justification

The view from the eastern end of the factory should show the chimney on the left and a window in the wall. The roofline should appear behind the chimney. Only figure D shows this.

## 4. Analysing and Modelling

## a). Data Necessity and Sufficiency

In this category, candidates are asked to determine what data is either necessary to contribute to the solution of a problem or sufficient, along with other data, to solve the problem. For example, a question could be of the form "Which of the following additional pieces of data would be sufficient to answer this question?". The candidate will not usually be asked to solve the problem.

## Sample question

Bill and Colin are twin brothers. They plan to meet at a hotel between their houses to exchange birthday presents. Bill's journey is on motorways and he travels at an average $120 \mathrm{~km} / \mathrm{hr}$. Colin's journey is on minor roads and he travels at $80 \mathrm{~km} / \mathrm{hr}$. Bill leaves at 10 am . They expect to arrive at the hotel at the same time.

Which one of the following further pieces of information would be sufficient to determine at what time they will meet?

A Colin's journey time
B Colin's travel distance
C Colin's departure time
D Bill's travel distance
E The difference in time between their leaving

## Key D

## Stimulus Type Verbal

## Justification

In order to find the meeting time, from information on either journey, the following calculation has to be carried out:

$$
\text { Meeting time }=\text { departure time + travel distance / average speed. }
$$

For Bill, we have two of the values on the right hand side (departure time and average speed), for Colin we have only one (average speed). Thus, the only single piece of information which would be sufficient to calculate the meeting time would be Bill's travel distance, so D is correct.

## b). Choosing a Model

A model of a set of data may be in the form of a graph, a diagram or a verbal description. Candidates are presented with a scenario which incorporates information on the way in which a parameter or a process varies. They are then asked to choose a representation of this variation in another form which models the data presented. A more difficult question might expect the candidate to derive a mathematical model and use it to produce an answer. The skill is to recognise how one set of information may model either another set or a real situation.

## Sample question

The drawing shows a cross section of a bottle used in a laboratory. In order to determine the volume of liquid in the bottle for various heights, water is allowed to flow into the bottle at a constant rate. An observer makes frequent notes of the height the water has reached and of the time. He draws a graph to illustrate his observations.


Which of the following graphs could be the one drawn by the observer?


## Key D

Stimulus Type Verbal and Pictorial

## Justification

The height will rise quickly at first (as the vessel is narrow at the bottom), it will then rise more slowly as the level reaches the wider section and more quickly again at the narrower section towards the top. All these segments will show curves as the area changes continuously. The rate of rise will then become constant in the cylindrical section at the top, giving a straight line segment. $D$ is the only graph to show this.

## c). Choices and Decisions

Candidates are given data in verbal, tabular, graphical, pictorial or diagrammatic form and a set of criteria. They are then asked to make a choice from a set of options based on the given data and criteria. This will involve a search of some sort. In the simplest sense, this would be just extracting an item from the data set (and would be equivalent to type 1a) or the search may involve an amount of data handling. The skill is to be able to make an informed decision by combining a set of information and given criteria for selection.

## Sample question

I have none of my nephew's favourite biscuits left. My nephew visits me, without fail, at least 3 times a week, though never more than five times. On each visit I know he will eat at least 6 biscuits but I won't let him have more than 8 . Packets of biscuits can contain as few as 10 biscuits, or as many as 12 .

How many packets of biscuits must I buy to make sure I do not run out within the next two weeks?
A 3
B 4
C 5
D 6
E 8

## Key E

## Stimulus Type Verbal

## Justification

A decision on how many to buy is made by dividing my nephew's maximum requirement by the minimum number of biscuits in a pack. This is the only way of ensuring there are enough.

The most biscuits my nephew will eat in two weeks is 2 (weeks) $\times 5$ (maximum visits per week) $\times 8$ (maximum number of biscuits per visit) $=80$. The minimum number of biscuits in a pack is 10 , so 8 packets will be required. E is correct.

### 5.2 Critical Thinking

## SKILLS ‘CONTENT’

This section tests the candidate's ability to use the following skills:

- Recognise reasoned arguments
- Identify conclusions
- Draw conclusions
- Recognise implicit assumptions
- Assess the impact of further evidence or information
- Recognise flaws in reasoning
- Select plausible explanations
- Recognise the logical functions of key elements
- Understand and/or clarify key terms and expressions


## TYPES \& EXAMPLES OF QUESTIONS

The questions in Paper 1 are all in multiple choice format. There are three types of question:

- Standard multiple-choice. A short stimulus is provided. The task is to select the most appropriate response from five options A-E.
- Assessing a response. The candidate is supplied with both a stimulus and a response, for example the response might be a counter-argument, objection, interpretation, etc. The candidate is required to assess the suitability of the response in relation to the stimulus by selecting the most appropriate of five given multiple choice options.
- Completion task. Part of the stimulus, such as a key phrase, is left blank. The task is to select the option which most logically completes the reasoning in the stimulus.

There are a number of different stimulus types for the multiple choice questions:

- A complete reasoned argument (possibly containing flaws)
- A passage with no stated conclusion but from which a conclusion can be drawn
- Factual items (e.g. statistics, observations, outcomes)
- Graphical or numerical data, with or without accompanying text
- A passage with a missing element (for completion tasks)
- A passage followed by a comment or response (e.g. counter argument, objection)
- A definition

The stimulus materials are drawn from a wide variety of sources: newspaper articles, editorial comments, letters and e-mails, science reports, everyday conversations, the Internet, debates, and so on. They are edited to ensure conciseness, clarity and simplicity of language.

Each of the first 9 sub-skill categories for Critical Thinking is explained below. An example of a question that tests the sub-skill is also provided.

## 1

Recognising Reasoned Arguments
Candidates are asked to distinguish between short arguments (of various kinds) and non-arguments.

## Sample question

In an argument there must be a conclusion, and one or more reasons given to support the conclusion.
From the above definition, which of the following is most obviously an argument?
A We all learn to speak and understand our own language before we learn any rules of grammar. Some people don't know what grammar is until they start learning a second language.
B It is becoming less and less important to teach spelling and grammar at school. Word processors can already correct most spelling and grammatical mistakes and the technology is improving all the time.
C Spelling mistakes are not a sign of being unintelligent. My sister makes lots of spelling mistakes; but then, my sister is not very intelligent.
D Spelling is even less important than grammar. Anyway, people who insist on grammar and spelling being exactly right need something more worthwhile to occupy their minds.
E There are two main kinds of spelling systems. In one system each letter or group of letters is always pronounced the same way; in the other system there can be many different pronunciations. English is of the second kind.

## Key B

## Question Type Standard

Stimulus Type Definition

## Justification for B

The claim that teaching spelling and grammar is becoming less important is supported by the two claims that (1) technology can already correct most mistakes and (2) that it is improving. Thus, whether you accept the conclusion or challenge it, B is an argument according to the definition. None of the other responses comply.

The argument form can clearly be seen by rearranging $B$ as follows:
'Word processors can already correct most spelling and grammatical mistakes and the technology is improving all the time. So it is becoming less and less important to teach spelling and grammar at school.'

The most basic skill of argument analysis is identifying the main conclusion and distinguishing it from the reasons (premises) that support it. Candidates are given arguments of different kinds and asked to select a sentence which is the stated conclusion, or a close paraphrase of the main conclusion. In some arguments the conclusion is not contained in one discrete statement. For such arguments the task will be to select the best expression of the overall conclusion of the passage.

## Sample question

Cycle lanes, which are painted on the road, are at best useless, and at worst more dangerous to cyclists than no lanes at all. The reason for having bicycle lanes on busy roads is to protect cyclists from injury by passing cars. For this objective to be met, there has to be a raised kerb or physical barrier that separates the cyclist from the main road, to prevent drivers from straying into the cycle lane, or blocking it with their parked vehicles. Painted lines do not prevent either of these abuses by drivers: all they do is give cyclists a false sense of security, which may make them less vigilant.

Which one of the following expresses the main conclusion of the above argument?
A Painted cycle lanes give cyclists a false sense of security and make them less vigilant.
B Cycle lanes do not protect cyclists from injury by passing cars unless they are separated from the main road.
C Painted cycle lanes are useless, and may even increase the danger to cyclists.
D Drivers have to be prevented from straying into cycle lanes or parking their cars on them.
E Only by placing a kerb or barrier between cycle lanes and the main road can cyclists' safety be assured.

## Key C

## Question Type Standard

Stimulus Type Complete Reasoned Argument

## Justification for C

The first sentence in the passage is the conclusion: that painted lines are useless and even dangerous. What follows it consists of reasons for this claim: that lines don't protect cyclists and do make them less careful. C is a close paraphrase of the first sentence.

The other responses all express reasons or assumptions that support the conclusion.

## 3

Drawing Conclusions
When presented with information or evidence, it is important to be able to determine what can be justifiably inferred from it, and what cannot. Candidates are asked to select a conclusion, inference or hypothesis which is supported by the content of the stimulus. The stimulus material may be in text, numerical or graphical/pictorial form.


Which of the statements below is a conclusion which follows from the above data?
A Having more police on the streets has reduced crime but at the same time caused more public anxiety.
B More police officers and less actual incidents have not led to a reduction in public anxiety about crime.
C Higher numbers of police on the streets has made people more anxious, even though crime rates have fallen.
D The public are not convinced by statistics when it comes to emotional issues.
E The police are too busy fighting real crime to worry about public anxieties.

Key B
Question Type Standard
Stimulus Type Graphical Data
Justification for B
It can be seen from the first two graphs that policing levels have risen and crime rates have fallen. But it can be concluded from the third graph that neither of these factors has reduced public anxiety (which has risen). B can be inferred because it does not jump to any conclusions about the reasons for these figures.

A and $C$ assume that the rise in policing level has caused one or both of the other two trends, for which there is no evidence in the data. D offers a different explanation for which there is no evidence either. E makes a related observation but one which neither follows from, nor helps to explain, the data.

## Recognising Implicit Assumptions

In many, if not most, everyday arguments, certain premises are left out, perhaps because they 'go without saying', or because it suits the author to omit them. In Critical Thinking they are termed assumptions (or implicit assumptions or underlying assumptions) as opposed to stated reasons. Recognising such assumptions is an essential skill in analysing and evaluating argument, because if the assumptions are false or questionable, the argument is undermined. Candidates are asked to identify which of a range of given statements, not explicit in the text, is nevertheless necessary to ensure the soundness of the argument.

## Sample question

Since there has to be some limit on the funding of university education by governments, it would be best to target such funding, giving a lower priority to subjects which provide little benefit to society. Some subjects, such as medicine, engineering and computing should be well funded, because they are clearly of great value to the community. Subjects such as history and art should be funded at a much lower level. These 'hobby' subjects are primarily of interest to the individual, and whilst a wealthy state should give some subsidy to them, it must be recognised that the future of society lies in the training and development of those people who will contribute the most.

Which of the following is an assumption underlying the above argument?
A Medicine, engineering and computing are not very interesting subjects to study.
B The study at universities of history and art provides little benefit to society.
C Subjects which are of interest to individuals can also provide benefit to society.
D The state should not subsidise subjects which are of interest only to individuals.
E The training of those people who will contribute most to society requires an increase in the government's funding of university education.

## Key B

Question Type Standard
Stimulus Type Complete Reasoned Argument Justification for B

The argument is that lower funding should be given to subjects such as history and art, which are primarily of interest to the individual. The grounds for this conclusion are that priority should be given to subjects that benefit society. Clearly for the conclusion to follow it must be assumed that history and art provide little benefit to society, a proviso expressed by B.

There is no need to assume $A$ : interest does not come into it. C is not assumed because it runs counter to the argument. D is not assumed because the passage states that some funding should be available to 'hobby' subjects. E need not be assumed because the argument is about targeting present funding, not increasing it overall.

## 5 Assessing the Impact of Further Evidence or Information

Someone may respond to a particular argument by presenting additional evidence or information. So it is important to be able to assess the impact of such information on the argument. Candidates are asked to select the most appropriate claim or item of evidence which would, if true, either strengthen or weaken the argument. Alternatively, they are presented with one response to the stimulus and asked to assess its impact on the argument.

## Sample question

Car drivers should have to pay for driving through major cities. This would be a fair policy, because most car drivers can afford to pay, and poorer people usually travel by bus. The principal aim of traffic policies for major cities should be to solve the problem of congestion. The more cars there are in cities, the more pollution there is, and the longer are journey times for everyone. Thus car drivers benefit from the comfort of car travel whilst other people suffer. The money raised from this charge could be used to make city centres look more attractive, which would benefit everyone.

After hearing the argument, one listener made the following comment:
'Many car drivers would not drive their cars in cities if they had to pay for doing so.'
If true, does the comment:
A Strongly support the conclusion of the argument?
B Give only a little support to the conclusion of the argument?
C Neither strengthen nor weaken the argument?
D Slightly weaken the argument?
E Seriously undermine the argument?

## Key A

## Question Type Assessing a Response

Stimulus Type Passage followed by a Comment

## Justification for A

The main premise of the argument for drivers having to pay is that the aim of traffic policies should be to solve congestion. By predicting that many drivers would be deterred by the charge, the comment strongly supports the conclusion.

## 6

Recognising Flaws in Reasoning
Candidates are asked to identify various faults or errors in arguments where the reasons - even if true - do not adequately support the conclusion.

## Sample question

Mountaineering is a very hazardous sport, and it takes many years of experience to learn how to survive in mountainous regions. The weather can change abruptly when you are a few thousand feet above sea-level, sometimes with disastrous consequences. Even if you know what signs to look for, and what to do if violent weather descends, you still run life-threatening risks. Therefore someone without knowledge and experience has no chance of survival if they are caught in a bad storm at high altitude.

Which of the following identifies the main flaw in the argument?
A Even at sea level, bad weather can have disastrous consequences.
B Even experienced mountaineers can have fatal accidents.
C Even inexperienced mountaineers can have lucky escapes.
D Even in good weather, climbing high mountains can be dangerous.
E Even weather experts can make totally incorrect forecasts.

## Key C

## Question Type Standard

Stimulus Type Complete Reasoned Argument (containing a flaw)

## Justification for C

The fault in the reasoning is that it draws much too strong a conclusion. The dangers that face even experienced climbers do imply that an inexperience person has a poor chance of survival in a highaltitude storm, but not that such a person has no chance. C points out one of the reasons why the conclusion goes too far.
$A, B$ and $D$ all lend weight to the argument, rather than exposing any deficiency. E misses the point of the argument.

## 7 Recognising Plausible Explanations

Reasons do not always function as claims made in support of a conclusion; sometimes reasons function as explanations for a fact or observation. Explanations are particularly relevant where circumstances are puzzling, anomalous, etc. Candidates are asked to select from a range of choices a statement which offers a plausible explanation for a fact or set of data.

## Sample question

Measurements were taken showing the growth of 16 fir trees planted at the same time but at different altitudes on a hillside. The results were recorded as follows.


Which of the following, if true, could be an explanation for the results shown by the graph?
A The height of a tree can be used to estimate the altitude at which it is growing.
B The trees growing at lower altitudes were taller than trees on high ground.
C Soil quality at higher altitudes gets progressively poorer.
D Fir trees can grow at altitudes at which other trees cannot survive.
E At a certain altitude trees stop growing altogether.

## Key C

## Question Type Standard

Stimulus Type Graphical Data

## Justification for C

The data shows a general tendency for trees to be shorter the higher up the hillside they are growing. If soil quality were found to be poorer with altitude, this could be the reason for the findings.

Response A, if true, might follow from the findings, but does not help to explain them. B merely states what the data shows. Responses $D$ and $E$ are facts that relate to tree growth, but not to the way tree height reduces with altitude.

## 8

Recognising the Logical Functions of Key Elements
Texts which present arguments have certain key elements, most obviously reasons (premises) and conclusions, including intermediate conclusions. There are also more special functions such as: example, counter-example, analogy, alternative explanation, general principle, etc. Candidates are asked to identify or assess the part certain words, phrases, or sentences play in a reasoned argument. Alternatively, they are asked to supply a key element which is required to compete a particular piece of reasoning.

## Sample question

Clones are identical - aren't they? They have the same genes, and therefore they have exactly the same characteristics as the parent organism. That is what we are told in science fiction, anyway. And that is what we are told is the chief reason for attempting to produce clones. But the first suggestion that the science fiction writers have got it wrong came last year with the birth of CC, a cloned tortoiseshell cat $\qquad$ . So, if you thought having the same genes makes clones behave or even look alike, think again!

Which of the following phrases, inserted in the blank space, most logically completes the passage?

A in a Cambridge laboratory.
B with identical genes to its parent.
C with different markings from its parent.
D which was the first of its kind.
E which looks and behaves like any normal cat.

## Key C

Question Type Completion
Stimulus Type Passage with Missing Element
Justification for C
The conclusion reached here is that we need to rethink our understanding of clones as being identical to their parents. The obvious premise for this will be that in some cases there is a difference between a clone and its parent, for instance different markings. C completes such a premise.

The other responses would fit the sentence grammatically, but not logically. Responses A D and E are immaterial. Response B would imply the opposite conclusion: that we do not need to rethink our understanding of clones.

Often the assessment of an argument depends on how certain key terms or expressions in the text are understood. This can be especially so when the expressions in question are either vague or ambiguous. Sometimes the authors of arguments 'help themselves to definitions' and it is important to recognise this. Candidates are asked to clarify the meanings of key words, phrases or sentences.

It is also important to understand the nature and the effect of claims made in an argument: for example, whether they are factual or judgemental, general or particular, strong or weak, significant or irrelevant, conditional or unconditional. Candidates are asked to show that they can recognise some of these differences.

## Sample question

If the media give publicity to certain types of crime, it may encourage criminals to carry out 'copy cat' offences. If, however, they were forbidden to divulge details of crimes, this would amount to censorship. The freedom of speech of the media is too important to sacrifice, so the media should be free to report certain crimes even if this means some other crimes are committed which would not otherwise be committed.

After hearing the above argument, a student summarised the conclusion as follows:
"There should be no restriction on the media's reporting of crime."
Has the student:
A expressed the conclusion fairly and accurately?
B expressed the conclusion too strongly?
C not expressed the conclusion strongly enough?
D contradicted the conclusion?
E identified the wrong part of the argument as the conclusion?

## Key B

## Question Type Assessing a Response

Stimulus Type Passage followed by a Comment
Justification for B
The passage deals specifically with the problem of copy-cat offences and concludes that the media should be free to report such crimes even if it prompts some other crimes. This does not imply that there are no circumstances under which press freedom should be restricted; therefore the summary is expressed too strongly.

## 6. Paper 2: Critical Reasoning

## SKILLS ‘CONTENT’

Paper 2 broadly draws on the content of the Critical Thinking component of Paper 1. The principal difference between the papers is that in Paper 2 candidates construct their own answers, and present their own reasoning, instead of selecting from multiple choice options.

The main skills addressed in Paper 2 are: inference and deduction, analysis, evaluation and constructing argument.

## Inference and Deduction

When presented with evidence or information students should be able to recognise what conclusions follow from them, and, just as importantly, what conclusions do not. This is an important skill to develop, for example when drawing conclusions about causal explanations, blame or responsibility.

Students should also be aware of the degree of support their conclusions receive from the evidence or information given. They should understand the difference between knowledge and belief, and be able to express conclusions appropriately in terms of certainty, likelihood, possibility/impossibility.

## Analysis

In critical thinking, analysis means the identification of the key elements of a reasoned case; most importantly its premises and its main and intermediate conclusions. Detailed analysis may also reveal such features as assumptions (missing premises), counter-arguments, background information outside the main argument, and so on.

## Evaluation

Evaluation involves judging claims or arguments against certain relevant criteria. For example, it may mean assessing the reliability of a piece of evidence, or the soundness and effectiveness of a line of reasoning. Evaluation can involve looking beyond the argument, for example when candidates use a counter-example, alternative explanation, analogy, etc. to show a weakness. (In this respect there is some overlap between evaluation and further argument.)

## Constructing argument

As well as analysing and evaluating arguments, students are encouraged to present their own, relevant arguments, whether in opposition to, or in support of, the original case. Further argument can arise out of evaluation, for example by giving an additional example to support a step in the reasoning or it may be a different line of reasoning altogether. The skill of further argument is the ability to add something new, but at the same time remain within the scope of the original argument.

## TYPES \& EXAMPLES OF QUESTIONS

Paper 2 consists of 4 written response questions.

## Q1 Evaluating Evidence

The concept of 'evidence' has a special role in a wide range of reasoning activities. The paradigm example is in the application of the law, where there are strict rules determining what counts as evidence, and what can or cannot be concluded from it. But in all kinds of contexts, from scientific investigation to historical research to everyday decision making, the evaluation and 'weighing' of evidence plays a crucial part.

Candidates are presented with a scenario in which several items of evidence, of different kinds, are presented. The tasks are to assess these and/or their sources, and to come to reasoned judgements on the basis of them.

The kind of criteria candidates are expected to consider will include:

- the nature of the evidence - fact, opinion, eyewitness account, 'hearsay', circumstantial evidence, conclusive evidence ('smoking gun'), etc.
- where relevant, the reputation, authority or expertise of a witness;
- the ability of a witness to observe the details alleged in the evidence;
- the degree of independence of a source, e.g. from bias or vested interest;
- whether there is corroboration of a claim from independent sources, or from known facts;
- the plausibility of a claim, or whether the source can provide credible reasons for the claim.

The candidates' ability to approach the evidence fairly and objectively will be assessed through their answers to three questions requiring short written answers and on one question which asks them to draw a general conclusion, or reach a decision, based on their evaluation of the evidence.

## Sample questions

- What support is there for Jasbir's version of what happened?
- Is Maria a reliable witness?
- On the basis of the available evidence, whose story is more believable, Faruq's or Jane's?


## Q2 Understanding and Evaluating Argument

Candidates are presented with a short argument of around 350 words, which they are expected to assess using the four skill areas of Paper 2. There are four or five questions based on the passage requiring short answers.

## Sample questions

- How successfully does the author deal with the objection mentioned in the third paragraph?
- Explain why the example given in the second paragraph is particularly important to the argument?


## Q3

Reasoning in a Scientific Context
Candidates are presented with argument, evidence, or information in numerical, graphical or pictorial form, with or without an accompanying text. Whilst being broadly scientific in content and treatment, the material does not require specialist scientific knowledge or understanding.

Candidates are asked four questions, which require short written answers testing, for example, their ability to assess hypotheses, draw conclusions and suggest plausible explanations.

## Sample questions

- Does the evidence support the conclusion that....? Give a reason for your answer.
- Give two possible explanations for the difference between the figures in table $A$ and table $B$


## Q4 Evaluating and Presenting Argument

Candidates are presented with an argument of around 350 words. They are given four or five critical thinking tasks which they must complete in a single short essay answer. They must show that they understand argument structure; that they can they can comment critically on the soundness and effectiveness of the reasoning; and that they can extend the reasoning using ideas of their own.

## Sample questions

- Show that you have a clear understanding of the argument by identifying its main conclusion and the reasoning used to support it.
- Point out any assumptions that are required for the argument to succeed, and say whether or not they are reasonable assumptions.
- Evaluate the argument by discussing its strong points and/or its weaknesses or flaws.
- Offer two further reasons, or short arguments, which could be used for or against the argument (both 'for', both 'against', or one of each).


## 7. Paper 3: Problem Solving and Critical Thinking (Advanced)

### 7.1 Problem Solving

## SKILLS 'CONTENT’

The paper will test the same sorts of skills as identified in paper 1, i.e. the ten 'sub-skills' of problem solving.

The mathematical content for papers 3 and 4 will, in addition to junior/elementary school mathematics, require some knowledge of:

- Numerical probability (e.g. the throws of a die), including the combination of independent and non-independent events.
- Expressing problems as linear algebraic equations and the solution of these equations.
- Expressing problems as a pair of simultaneous equations with integral coefficients and the solution of these equations.
- Use and manipulation of integer powers (e.g. knowing that $3^{3} \times 3^{2}=3^{5}$ ) and square roots.
- The use of tree and decision diagrams.
- The calculation and use of mode, mean and median of sets of data.
- The use of Venn and Carroll diagrams for solution of categorisation problems.


## TYPES \& EXAMPLES OF QUESTIONS

The questions have short stimulus passages and will be in multiple choice format (requiring the selection of one or more correct answers). The stimulus types will be the same as for paper 1 (e.g. tabular, graphical, verbal, pictorial and/or diagrammatic).

Examples of questions using some of these additional methods are given below.

## Sample question (that uses probability)

I have five coins in my pocket: one 5 c coin, two 10 c coins, one 20 c coin and one 50 c coin. If I take two coins out at random, what are the chances that I will have enough to buy a 30c newspaper?

A $2 / 5$
B $\quad 1 / 4$
C $3 / 10$
D $3 / 5$
E $8 / 13$

## Sample solution:

This has the form of a single question and is worth 2 marks as quite a lot of working is needed.
A solution requires combined (and non-independent) probability calculations. One way of solving it is by using a tree diagram. This is set out below in the form of a table. The method may easily be extended to more than 2 items.

| First coin <br> drawn | Probability | Second coin <br> drawn | Probability | Total money <br> drawn | Combined <br> probability |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $5 c$ | $10 c$ | $1 / 2$ | $15 c$ | $1 / 10$ |  |
|  |  | $1 / 5$ | $20 c$ | $1 / 4$ | $25 c$ |

The correct answer is D.
The asterisked values show where there is enough drawn to buy the paper: 8 outcomes out of 13 in total. The calculations may be checked by ascertaining that the last column sums to 1 . The sum of the probabilities of the 8 successful outcomes is $12 / 20=3 / 5$.

Answer $A$ is the probability of drawing exactly 30 c with two coins $(1 / 10+1 / 10)$
Answer B correctly counts 8 ways of drawing 30 c or over but calculates the total number of ways of drawing two coins as $2^{5}=32$, all with equal probability.
Answer C calculates that there are 6 ways of drawing 30c or more ( $50 \mathrm{c}+$ anything - four ways and $20 c+10 c-2$ ways) out of $20(5 \times 4)$ total possibilities.
Answer E sums the number of satisfactory outcomes instead of the probabilities.

## Sample question (that uses algebra)

The Colorfone telephone company offers two quarterly tariffs:
Tariff L : $\quad$ 12c per unit for the first 500 units then 6.5 c per unit for any extra.
Tariff H : $\quad \$ 50$ rental and 4 c per unit for all calls.
Clearly Tariff $L$ is less expensive for light users.
(i) What are the relative costs of the two tariffs at a usage of 600 units per quarter?

A Tariff $L$ is $\$ 2.00$ cheaper
B Tariff $L$ is $\$ 7.50$ cheaper
C Tariff $L$ is $\$ 35.00$ cheaper
D Tariff H is $\$ 12.50$ cheaper
E Tariff H is $\$ 42.50$ cheaper
(ii) Above what quarterly usage in units is it advantageous to use tariff H rather than Tariff L ?

A 417 units
B 500 units
C 625 units
D 900 units
E 2000 units

## Solution/Comments

This item has two questions, each worth 1 mark.
(i) The correct answer is B.

At 600 units: Tariff $L$ costs $500 \times .12+100 \times .065=60+6.5=\$ 66.5$.
Tariff H costs $50+600 \times .04=50+24=\$ 74$
Answer A calculates tariff H correctly, but calculates tariff $L$ as $600 \times .12=\$ 72$ (forgetting the 6.5c rate).
Answer C calculates tariff H correctly but calculates tariff $L$ as $600 \times .065=\$ 39$ (forgetting the 12 c rate).
Answer D calculates tariff L correctly but calculates tariff H as $50+100 \times .04=\$ 54$ (forgetting that the
.04 c rate is on all calls).
Answer E calculates tariff L correctly but calculates tariff H as $600 \times .04=\$ 24$ (forgetting the standing charge).
(ii) The correct answer is $D$.

This differs from many AS level problems in that the intersection may be with either of two lines. A sketch graph would help but the problem can be solved quite easily with some logical thought to decide on which part of the line the intersection occurs.

If we look at the prices at 500 units' usage:
Tariff $L$ costs $500 \times 12 / 100=\$ 60$
Tariff H costs $50+(500 \times 4 / 100)=\$ 70$
So the point we are looking for must be above 500 units per quarter usage.

This could be solved by trial and improvement but it is quicker and we can be more confident in the answer if algebra is used. We can now form an equation. If $x$ represents the units used, we are looking for the point where the two tariffs are equal:

Tariff $L$ costs $\$ 60$ for the first 500 units, so at $x$ units the cost is:
$60+6.5(x-500) / 100$
Tariff H costs: $50+4 x / 100$.
We are looking for the point at which these two amounts are equal, so

$$
60+6.5(x-500) / 100=50+4 x / 100
$$

This may be simplified:

$$
2.5 x=2250
$$

So $x=900$. Above 900 units Tariff H is cheaper.
(An alternative and simpler solution requiring clear thought notes that the difference at 500 units is $\$ 10$ and above that the difference per unit is 2.5 c , so the intersection must happen at $1000 / 2.5=400$ units above 500 units. It is often possible to solve such problems without using algebraic notation.)

Answer A compares the 12 c rate on tariff $L$ with the $\$ 50$ standing charge on tariff $\mathrm{H}(5000 / 12)$
Answer B simply draws the answer from the 500 unit rate change in the stimulus.
Answer $C$ forgets the 12c rate on tariff $L$ and does all the calculations using the 6.5 c rate.
Answer $E$ forgets the 6.5 c rate on tariff $L$ and does all the calculations using the 12 c rate.

## Sample question

In the country of Numismia, the currency is the mart and coins exist in all denominations from 1 mart to 10 mart (i.e. 1 mart, 2 mart, 3 mart etc.) The next smallest is then 20 mart.

Ayesha has four coins in her purse, with three different denominations, adding up to 13 mart. Boris also has four coins in his pocket, with the same three denominations, but they add up to 17 mart.

Boris has two of one denomination of coin. There are two possibilities for what this could be. Select them both.

A $\quad 3$ mart
B $\quad 4$ mart
C $\quad 5$ mart
D 6 mart
E 7 mart

## Solution/Comments

If the candidate selects exactly two correct answers they score two marks.
If they select one correct and one wrong answer they score one mark.
If they select three or more answers they score zero marks.
The correct answers are C and D.
Boris has either two 5 m coins $(1+5+5+6)$ or two 6 m coins $(2+3+6+6)$.
If we look at the ways of partitioning 13 into 4 integers with two the same (the contents of Ayesha's purse) we have the following possibilities:

$$
\begin{array}{lllllllllll}
1129 & 1138 & 1147 & 1156 & 1228 & 1336 & 2236 & 2245 & 2335 & 1255 & 2344
\end{array}
$$

This gives the following as the denominations they both have:

| 129 | 138 | 147 | 156 | 128 | 136 | 236 | 245 | 235 | 125 | 234 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

We need to find which of these, with a single repeated coin, could add up to 17. There are only two possibilities

$$
\text { 236: } 2+3+6+6=17 \quad \text { and } \quad 156: 1+5+5
$$

In the first case, Boris has 25 m coins and in the second he has two 6 m coins.
The distractors are other coins which (with others) can make up either 13 m or 17 m .

### 7.2 Critical Thinking

## SKILLS ‘CONTENT’

The paper will test the same sorts of skills as identified in paper 1, i.e. the sub-skills of critical thinking.
Candidates are tested on their ability to analyse, evaluate and respond appropriately to more complex arguments; and to draw reliable inferences from written texts and other forms of information.

## TYPES \& EXAMPLES OF QUESTIONS

The questions will be both more varied and searching than those in Paper 1. Often candidates will be required to draw on more than one skill in order to make the correct response. They may also be asked to justify a particular response by indicating why it is the right answer and others are wrong.

The questions will each be based on a stimulus material, sometimes with one stimulus item supporting two or more questions. These items are drawn from a wide variety of subjects and sources: articles, editorials, letters and e-mails, reports, Internet sites, etc. Although the material will be challenging, it will be selected and edited with a view to its accessibility to candidates who have a good command of English, but not necessarily as their first language. Candidates will be required to select one or more correct answers.

## Sample question

There is a growing debate about which sports should be represented in future Olympic Games. Many high-profile sports already have their own regular world championships which are more important and prestigious than the Olympics. For example, the football World Cup, the Wimbledon tennis tournament or cycling's Tour de France mean far more to the players and fans of those sports than the corresponding Olympic event, in which some of the star performers show little or no interest. Only sports in which an Olympic Gold Medal is seen as the supreme achievement, should be included in the Games. Any plans to add golf at the next Olympic sports should therefore be shelved without further discussion.

If you knew nothing about golf, which ONE of the following would you need to verify before accepting the conclusion of the argument?

A Golf is scheduled for a place in the next Olympic Games.
B There is at least one regular world golf championship.
C If golf is included in the Olympic Games, some less popular sports will have to be excluded.
D The popularity of golf is equal to that of tennis, football or cycling.
E A significant number of countries that compete in the Olympic Games have golf courses.

## Solution/Comment

The correct answer is $B$.
The passage argues that certain events should be left out of the Olympics on the grounds that they have their own regular world championships. If this argument is to apply to golf - as the conclusion recommends - it would be a minimum requirement that golf has at least one such event. None of the other conditions would be necessary for the conclusion to follow.

## Sample question

There is evidence that birds of prey such as buzzards and black kites prefer to hunt their prey near roads rather than in the open countryside. Furthermore, they prefer to hunt near busy roads rather than quiet roads. Their preference can be explained in a number of ways. For one thing a busy road like a motorway provides a steady stream of fast-moving traffic which the birds adjust to more easily than to the sight and sound of an occasional vehicle on quiet roads. For another motorways have verges which provide open areas for hunting and shrubs at convenient perching height. It would seem that ever-expanding road-building programmes are, contrary to environmentalists fears, good news for wildlife.

Statement: Birds of prey are just a narrow range of species with particular needs and preferences.
Which ONE of the following best describes how the statement relates to the passage?
A It is an assumption on which the argument depends.
B It strengthens the argument.
C It provides an explanation of the evidence.
D It exposes a flaw in the reasoning.
E It follows logically from the conclusion.

## Solution/Comments

The correct answer is $D$. The fault in the arguments is that it argues from a small range of cases to a very general conclusion. There are no grounds in the passage for concluding that wildlife as a whole will be better off. D exposes this flaw.

## 8. Paper 4: Applied Reasoning

## SKILLS ‘CONTENT’

Paper 4 assesses a candidate's ability to apply problem solving and critical thinking skills to real life situations which may be encountered in higher education and/or in employment. In addition to the skills defined under papers 1 to 3 , paper 4 tests two additional problem solving skills and the ability to construct a reasoned case.

The additional skills assessed in paper 4 are as follows:

## Develop a model or carry out an investigation

Modelling requires the candidate to develop a set of equations, a table or some other set of data which represents the mathematics of a real situation. As a very simple example, a telephone bill which consists of a standing charge plus a fixed rate per time of calls could be modelled as: $b=s+a t$, where $b$ is the bill, $s$ is the standing charge, a the cost per minute and $t$ the minutes used.

Investigation requires the candidate to take a real physical situation and extend the data and parameters given to draw conclusions about other related and possibly more complex situations.

## Analyse complex data and draw conclusions

Analysis requires the candidate to combine raw data in appropriate ways to enable the drawing of conclusions. The data may be presented in written, tabular and/or graphical form. The candidate is expected to select the pieces of data which are relevant to the questions asked, to look for patterns in the data and to analyse the data mathematically in an appropriate way to lead to the conclusions required.

## Constructing a Reasoned Case

Candidates are presented with three or more documents, of varying length and style that take differing perspectives on an issue. They are asked to draw on these materials to produce a reasoned case for or against a given viewpoint, decision or policy; or in answer to a specific question or questions.

The construction of a reasoned case will require candidates to:

- select relevant material from the documents presented
- synthesise information or opinion from different sources
- present a clear conclusion, decision or recommendation
- justify the conclusion, decision or recommendation with sound reasoning
- identify and respond to objections or alternative arguments.

TYPES \& (PARTIAL) EXAMPLES OF QUESTIONS

## Sample question (developing a model)

You are the engineering manager of a car racing team. Given the following information about fuel consumption, tank capacity, speed in relation to weight of fuel carried, average pit-stop time, etc., would you make 2 pit stops during the race or 3 ? Consider all the factors that need to be taken into account; justify and explain your decision.

## Sample question (investigation)

You are a consultant employed by the Bolandian government to make recommendations on a new decimal currency system. In particular, they wish to know what denominations of coin to issue in order to minimise the number of coins required for any transaction from 1 c to 99 c . Consider the most efficient denominations for a system having up to 5 different values of coin.

## Sample Question (drawing conclusions from complex data)

You are a scientist carrying out a study of the effects that various fertilizers have on crop yields. Given the results on experiments using several fertilizers at different application levels, the value of crops and the cost of the fertilizers, come to conclusions on the effectiveness of various fertilizers and their economic value.

## Sample question (constructing a reasoned case)

Identify the response made in Document 3 to the principal claim made in Paragraph 2 of Document 1. Does the principle stated in the first sentence of Document 2 support the recommendation made in the last paragraph?

## Sample question (constructing a reasoned case)

On the basis of Documents 1 to 4, construct a reasoned case either for or against adopting the policy recommended in Document 3.

## 9. Grade Descriptions

The Grade A candidate:

- has shown clear understanding of both simple and complex arguments in terms of their structure and techniques
- has shown consistent success at solving (often complex) problems requiring initiative, precision and attention to detail
- has demonstrated the ability to make perceptive critical evaluations of arguments in terms of their strengths, weaknesses, flaws, implicit assumptions, persuasive devices, etc.
- has demonstrated consistent ability to make sound inferences (e.g. from evidence); and to critically assess the reliability of claims and their sources
- has presented cogent, relevant and well-developed arguments in response to stimulus materials and questions
- has structured work so that the process of thought is made clear
- has demonstrated understanding of the conventions of language; and uses the vocabulary of reasoning appropriately

The Grade B candidate:

- has shown understanding of the broad structure of arguments, some of them complex
- has been successful in solving a number of problems requiring initiative and attention to detail
- has demonstrated the ability to evaluate arguments in terms of their soundness or effectiveness, making some reference to flaws, assumptions, etc.
- has demonstrated the ability to make generally sound inferences, and an awareness of the criteria needed to assess the reliability of claims and their sources
- has presented some sound, relevant arguments in response to stimulus materials and questions
- has generally made his or her thinking clear
- has demonstrated some understanding of the conventions of language; and on occasions used the vocabulary of reasoning with understanding of its correct usage

The Grade E candidate:

- has shown ability to recognise some of the characteristic features of reasoning
- has shown some success in solving problems
- has offered some evaluation of arguments in terms of their effectiveness
- has demonstrated some ability to draw conclusions from texts and other sources
- has offered some further arguments in response to stimulus materials and questions
- has constructed responses with sufficient clarity for meaning to be clear.


## 10. Resource List

## Recommended Text

Butterworth, J. and Thwaites, G. Thinking Skills Cambridge University Press, 2005, ISBN: 0521521491

## Suggested Background Reading

Van den Brink-Budgen, Roy Critical Thinking Publisher: First and Best 1995 ISBN:18608319105
Van den Brink-Budgen, Roy Critical Thinking for Students Publisher: How to Books Ltd (Edition 2000) ISBN: 1857036344

Bowell, T. and Kemp, G. Critical Thinking: A Concise Guide, Routledge, London, 2002

Barry, V.E. and Rudenauer, J.E. Invitation to Critical Thinking, Harcourt Grace
Copi, I.M. and Burgess-Jackson, K. Informal Logic, 2nd edition, Macmillan, New York, 1992
Ennis, R.H. Critical Thinking, Prentice Hall, 1995
Fisher, A. Critical Thinking: An Introduction, Cambridge University Press, Cambridge 2001
Govier, T. A Practical Study of Argument, Wadsworth, Belmont, CA, 1985
Paul, R.W. and Elder, L. Critical Thinking, Pearson Education 2002
Phelan, P. and Reynolds, P. Argument and Evidence: Critical Analysis for the Social Sciences, Routledge, London,1995

Thomson, A. Critical Reasoning in Ethics: A Practical Introduction, Routledge, London

Thomson, A. Critical Reasoning: A Practical Introduction, 2nd edition Routledge, London, 2002
Warburton, N. Thinking from A to Z, Routledge, 1996 ISBN: 0415096863
Weston, A. A Rulebook for Argument, Hackett
ICT and Internet-based Resources
www.criticalthinking.org
www.sjsu.edu/depts/it|
www.teachthinking.com

Copies of syllabuses, past papers and Examiners' reports are available on CD-ROM and can be ordered using the Publications Catalogue, which is available at www.cie.org.uk under 'Qualifications \& Diplomas' - 'Order Publications'.

