ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates should be reminded to read the questions carefully, ensuring that they are meeting the full demands of the question. Special attention should also be paid to the accuracy of a candidate's work. The rubric clearly states the accuracy required unless specified otherwise. This means that the working of solutions should be to a greater level of accuracy.

General comments

Many candidates performed well, showing a good understanding of the syllabus requirements and being able to apply the necessary techniques correctly. There did not appear to be any issues with timing and the majority of candidates set out their solutions in a clear and concise fashion.

Comments on specific questions

Question 1

Most candidates adopted the correct approach of forming a quadratic equation in x only and making use of the discriminant, obtaining the correct critical values. Errors occurred when attempting to give the correct range of values for a, with some candidates omitting to form inequalities. There were few errors in algebraic and arithmetic manipulation.

Answer: $a > 11$, $a < -5$

Question 2

Many completely correct solutions were seen, with candidates exhibiting a good understanding of indices.

Answer: $a = -\frac{13}{6}$, $b = 0$, $c = 1$

Question 3

The great majority of candidates correctly dealt with the change of base for $\log_{25} x$ and the application of the law of logarithms concerning indices, obtaining an equation involving $\log_5 x$. Many completely correct solutions were seen.

Answer: $x = 125$

Question 4

(i) There were some errors in the sketching of the graph of $y = |3 + 2x|$, with the vertex of the graph appearing on the y-axis rather than on the x-axis. The graph of $y = 2 - x$ was usually sketched correctly. Candidates should realise that it is good practice to mark in the numerical values of the x- and y- coordinates for the intercepts on their graphs.

(ii) Most candidates chose to make use of two separate linear equations in order to solve the given equation. Solutions using a quadratic equation formed by squaring both sides of the given equation were seldom seen. There were many correct solutions seen.

Answer: **(ii)** $x = -\frac{1}{3}$, $x = -5$

Question 5

It was evident that candidates are well practiced in distinguishing when to use permutations and when to use combinations.

(a) (i) Most candidates obtained the correct permutation in the form ${}^9P_6$ and went on to evaluate this correctly. However, there were quite a few candidates who did not obtain the correct evaluation.

(ii) Most candidates realised that a product of 3 permutations was needed, producing a correct response.

(iii) Candidates needed to realise that there were two different cases to consider as the symbols at the start and finish of the password could be interchanged. Provided this was done and the fact that again use of permutations was the best approach, many candidates obtained the correct answer.

(b) The majority of candidates realised that this part of the question involved combinations and produced correct answers for both parts.

Answers: **(a)(i)** 60480 **(ii)** 144 **(iii)** 1680 **(b)(i)** 2100 **(ii)** 420

Question 6

(i) It was essential that candidates used the correct notation to write down the range of the given function. Far too many responses were of the type $x > 6$, rather than the acceptable $\mathrm{f}(x) > 6$ or $y > 6$.

(ii) Most candidates were able to find the inverse of the given function giving their final answer using the correct notation. Some candidates did not state the range and domain of this inverse function and many of those that did used incorrect notation as in **part (i)**. The relationship between the domain and range of a function and its inverse was clearly not understood by many candidates.

(iii) Most candidates recognised the notation and made the appropriate differentiation.

(iv) Many correct solutions were seen with candidates obtaining $\frac{1}{4}\ln 2$, but then going on to give a rounded decimal answer. Those that did this were not penalised on this occasion, but candidates and centres should be aware of the meaning of the request for an exact answer.

Answers: **(i)** $\mathrm{f}(x) > 6$ **(ii)** $\mathrm{f}^{-1}(x) = \frac{1}{4}\ln(x-6)$, $x > 6$, $\mathrm{f}^{-1}(x) \in \mathbb{R}$ **(iii)** $4\mathrm{e}^{4x}$ **(iv)** $\frac{1}{4}\ln 2$

Question 7

(i) Provided candidates were able to obtain the correct relationship between the two remainders, correct values for *a* and for *b* were usually obtained. Candidates were expected to show that $a = 6$ by solving two simultaneous equations. A check that $a = 6$ in both simultaneous equations leading to the correct value for *b* was also an acceptable method.

(ii) Candidates using synthetic division gave answers that were too great by a factor of 2. Care should be taken when using this method. It was intended that candidates use observation or algebraic long division for this part of the question.

(iii) Candidates were given credit for an attempt to factorise their quadratic expression from **part (ii)**. Some candidates chose to carry on and give solutions for the expression equated to zero. These were not penalised provided the linear factors were written down.

Answers: **(i)** $b = 2$ **(ii)** $(2x-1)(3x^2+5x-2)$ **(iii)** $(2x-1)(3x-1)(x+2)$

Question 8

(i) Most candidates were able to find the gradient of the given graph, together with the intercept on the vertical axis. There were some errors when applying the logarithmic form of the given equation, allocating the values of *A* and of *b* incorrectly.

(ii) A correct approach either using the given graph or the equation found in **part (i)** was applied by most candidates.

(iii) A correct approach either using the given graph or the equation found in **part (i)** was applied by most candidates.

Answers: **(i)** $A = 27.5$, $b = 1.2$ **(ii)** 3.84 **(iii)** 113 or 112

Question 9

(i) Very few incorrect answers were seen, with most candidates applying the given relationship between the two areas correctly. It was hoped that candidates would obtain and work with $\frac{\pi}{4}$, rather than a decimal equivalent. If a decimal equivalent was used, it needed to be to more than two decimal places.

(ii) Most candidates applied a correct method, but did not give their solution to enough accuracy to start with to justify the given answer correct to 2 decimal places. It was expected that candidates state that, after their workings, $r = 7.180$, or $r = 7.1803$, either of which round to the given answer correct to 2 decimal places.

(iii) Many correct solutions were seen with a correct approach to the calculation of the perimeter being applied by most.

(iv) Many correct solutions were seen with a correct approach to the calculation of the required area being applied by most. There were some inaccurate answers given due to premature approximation in some cases. Candidates should ensure that they are working to the required level of accuracy throughout.

Answers: **(i)** $\frac{\pi}{4}$ **(iii)** 11.6 **(iv)** 1.11 or 1.08

Question 10

(i) Most candidates recognised the need to differentiate the given expression as a product, with very few errors being seen.

(ii) There were many very good attempts at this part, with many completely correct solutions seen. Candidates recognised integration as the reverse of differentiation and took note of the instruction 'Hence'. Most problems were not with making use of **part (i)** but with the algebraic simplification of a correct expression to the required form. Some candidates did not recognise that they needed to 'take out' a common factor of $(2x-1)^{\frac{3}{2}}$ from their two-term expression.

(iii) Provided their answer to **part (iii)** was in the required form, candidates were given credit for applying the limits correctly. It should be noted that the word 'Hence' is again used, meaning that the result from **part (ii)** must be used in this part. Some candidates chose to use the numerical integration function on their calculator in order to answer this part. This did not gain any credit.

Answers: **(i)** $3x(2x-1)^{\frac{1}{2}}+(2x-1)^{\frac{3}{2}}$ **(ii)** $\frac{(2x-1)^{\frac{3}{2}}}{15}(3x+1)$ **(iii)** $\frac{4}{15}$

Question 11

(i) Most candidates were able to produce a completely correct solution which was clearly set out and contained all the necessary steps.

(ii) Very few incorrect solutions were seen, with candidates making use of **part (i)** to solve a quadratic equation in $\tan\theta$. It should be noted that final answers, when in degrees, should be given correct to 1 decimal place.

Answer: **(ii)** $63.4^{\circ}, 123.7^{\circ}, 243.4^{\circ}, 303.7^{\circ}$

ADDITIONAL MATHEMATICS

Paper 0606/22

Paper 22

Key messages

In order to do well in this paper, candidates need to demonstrate that they are able to apply their skills. They need to use their knowledge of concepts and apply appropriate techniques to solve problems. Candidates need to read each question carefully and answer fully. This includes taking note of instructions such as “Do not use a calculator in this question” or “You must show all your working”. Whilst calculators are a useful tool and candidates should be encouraged to use them to check their solutions throughout, efficient use of a calculator is not an assessment objective of the syllabus. Candidates who omit to show key method steps in their solution to a question, through using a calculator, are generally not awarded full marks. Candidates should take care to ensure that, when solving problems involving angles, their calculator is set to the appropriate mode to answer the question correctly.

General comments

Many candidates gave well-presented and clearly thought-out solutions. Most candidates were clearly very well prepared for the examination. The level of algebra was a little challenging in places. Many candidates demonstrated good manipulative skills. This was exemplified in **Question 11**. Some candidates could improve by understanding that their working must be detailed enough to show their method clearly, with each key step being shown as this can allow method marks to be awarded; it is essential if a question asks candidates to “Show that…” This indicates that the answer has been given to the candidates and that the marks will be awarded for showing how that answer has been found. The need for this was highlighted in **Question 9(ii)** and **Question 12(i)**. Showing clear and full method is also very important if the use of a calculator is not allowed, such as in **Question 6**. There were no rounding issues in this paper, although some candidates omitted to take notice of the word “exact” in **Question 9(iii).** Candidates would do better if they realised that answers that are the full length of their calculator display are very unlikely to be exact. As the angle was measured in radians, the angle should have been given as a multiple of π. No candidate seemed to be short of time.

When answers are written in alternate locations within their script, candidates should indicate where they have written the continuation of their solution.

Comments on specific questions

Question 1

This question was well-answered by the majority of candidates. It proved to be a good, strong start to the question paper.

(i) Many candidates rewrote the expression as $y = 5(x-9)^{-\frac{1}{2}}$ and used the chain rule for differentiating a function of a function. Those who did this were usually successful. Some candidates used the quotient rule or, on occasion, the product rule for differentiation. These were less successful. Some candidates used $\frac{\mathrm{d}(5)}{\mathrm{d}x} = 1$ and produced a complicated expression that they then attempted to simplify. Extra practice in identifying the most efficient method of solution may have helped these students.

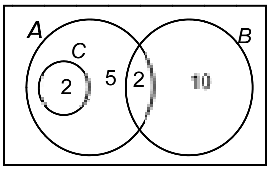
(ii) Most candidates understood the correct process – evaluating $\left.\dfrac{dy}{dx}\right|_{x=13}$ and multiplying the result by δx , which was h in this case. A small number of candidates omitted to multiply by δx or multiplied the expression for $\dfrac{dy}{dx}$ by h without evaluating it when $x = 13$.

Answers: **(i)** $\dfrac{dy}{dx} = -\dfrac{5}{2}(x-9)^{-\frac{3}{2}}$ **(ii)** $-0.3125h$

Question 2

Candidates demonstrated good skills in this area of the syllabus and answered this question well. Candidates needed to interpret the information given in order to complete the Venn diagram successfully and complete the question. Many candidates did this successfully. The large majority of students represented the sets A, B and C correctly in the Venn diagram. The numbers of elements in set B were usually correctly placed. Some candidates did not include the 2 elements that were in set C as elements of set A. These candidates commonly gave the answer for the number of elements in the set $A \cap B' \cap C'$ as 7.

Answer:



$n(A \cap B' \cap C') = 5$

Question 3

A small number of candidates attempted spurious integration of a product or quotient of terms. A few candidates evaluated $\dfrac{dy}{dx}$ when $x = 1$ and then found the equation of the tangent. The question asked for the equation of a curve. These candidates may have improved if they had read the question more carefully, therefore. The large majority of candidates understood that the correct first step was to rewrite the expression for the derivative as a difference of terms which were powers of x. The integration was usually well carried out and most candidates proved that they understood exactly what was required to solve the problem. A small number of candidates omitted to write y as the subject of the equation and usually something such as "equation = …" was given instead. These candidates may have improved if they had realised that the equation of a line or curve is only complete when quoted with its independent variable (x) and its dependent variable (y).

Answer: $y = 3x^3 + 3x^{-1} + 1$

Question 4

(a) Mostly well answered. Candidates who were able to interpret the equation and simply wrote down $a = 10$ were generally more successful than those attempted to use $\dfrac{\text{maximum} - \text{minimum}}{2}$. The value of b was usually well found. Some candidates converted to radians and then gave the value of b as $\dfrac{\pi}{3}$, misinterpreting the period of the function. Occasional arithmetic slips occurred when evaluating c. For example, it was not uncommon for c to be given as 14, with candidates seemingly thinking that $\cos 0 = 0$.

(b) Some candidates sketched graphs of the correct shape, over 2 cycles and gave some indication of the vertical scale they had used. Many fully correct solutions were seen. Candidates who either drew the line $y = -2$ or plotted the points $(0, -2)$, $(45, -2)$, $(90, -2)$, $(135, -2)$, $(180, -2)$ were usually more accurate than those who did not. Occasionally candidates sketched graph with minima at $y = -2$. The given domain was $0° \leqslant x \leqslant 180°$. Some candidates may have done better if they had observed this as they sketched more than the required two cycles of the function.

Answer: **(a)** $a = 10$, $b = 6$, $c = 4$

Question 5

(i) Most candidates understood how to apply the general binomial formula correctly and did so. Some candidates may have improved if they had taken note of the instruction to give each term in its simplest form. These candidates tended to leave the third term as $5103(kx)^2$.

(ii) The majority of candidates formed a correct equation and went on to solve it successfully. A small number of candidates misinterpreted the information, doubling the coefficient of x^2, rather than x. Some candidates equated terms rather than coefficients. This resulted in a spurious x component. These candidates would have improved if they had interpreted the word "coefficient" correctly.

Answers: **(i)** $2187 + 5103kx + 5103k^2x^2$ **(ii)** $k = 2$

Question 6

Candidates demonstrated good problem solving skills when answering this question. Answers were generally well set out and progressed logically. Most candidates constructed a correct proportional relationship. Most of these then proceeded to carry out a correct manipulation of terms to find the solution. The majority of candidates provided enough evidence of method for it to be clear that a calculator had not been used in the solution. Candidates who used $\frac{p+q\sqrt{3}}{6}$ rather than x in their calculations made the working more complicated than was necessary. Occasional arithmetic slips were made. A very small number of candidates attempted to find the area of each parallelogram. These candidates made no real progress towards a solution.

Answer: $p = -27$, $q = 23$

Question 7

(a) Candidates clearly understood the correct method of solution. Some candidates made sign errors.

(b)(i) Candidates who were careful with signs and arithmetic produced clear and accurate answers. Other candidates would have improved if they had taken more care with the arithmetic. For example, some candidates calculated $4 \times 0 = 4$ when finding the det **C**. A few candidates forgot to transform **C** by swopping the terms on the leading diagonal or changing the signs on other diagonal or both.

(ii) The simplest method of solution was to post-multiply **C** by **D**. Candidates who realised this usually arrived at the correct solution quite quickly. Occasional arithmetic errors were made. Those candidates who attempted to calculate the matrix product **DC** may have done better if they had fully understood that matrix multiplication in non-commutative. A small number of candidates chose to find $\mathbf{D}^{-1}$ and multiply by a general 2 by 2 matrix. This method of solution, forming and solving two pairs of simultaneous equations, whilst correct, was far longer. It was more likely that an error was made by candidates choosing this option.

Answers: **(a)** $\begin{pmatrix} -14 & 3 & 2 \\ -23 & 6 & 1 \end{pmatrix}$ **(b) (i)** $-\frac{1}{2}\begin{pmatrix} 1 & 0 \\ -4 & -2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} -8 & -6 \\ 13 & 7 \end{pmatrix}$

Question 8

(i) Most candidates provided excellent answers with full and clear solutions being given. Candidates who chose to use the substitution $x = 2y - 2$ had a more straightforward solution as the coefficients of the terms they were working with were integers. Candidates who chose to use $y = \frac{x+2}{2}$ needed to work with fractions. Occasionally, these candidates made sign errors. A very small number of candidates made no progress with this part of the question.

(ii) A small number of candidates stated the coordinates of the point $(-2, 0)$ as $(0, -2)$ in **part (i)** of this question. Whilst this usually was ignored in **part (i)**, it did affect the accuracy of the answer to **part**

(ii) Candidates chose to use Pythagoras' theorem and gradient relationships in roughly equal measure. Candidates were equally successful. A small number of candidates attempted to find the equations of the line segments that are the sides of the triangle *ABC* or found the area of the triangle.

Answer: **(i)** (–2, 0), (2, 2)

Question 9

(i) Some very good solutions were presented to this problem. More able candidates found the correct trigonometric ratios representing the sides *TR* and *RS*. Many candidates did this using simple ratios and definitions. Other candidates used appropriate trigonometric identities. A small number of candidates chose an incorrect ratio. Candidates who left their ratios in terms of sin θ and cos θ and tried to combine them as a single, compound fraction, often made sign errors.

(ii) Candidates found this part of the question more challenging. Candidates who wrote the sides *TR* and *RS* directly in terms of cosec θ and cot θ in **part (i)** made fewer errors in this part, presenting solutions that produced the requested result more easily. Candidates who wrote their answers to **part (i)** as three separate terms also arrived at the given result more swiftly.

(iii) There were a good number of correct answers given to this part. Candidates usually recognised the correct process and applied it. Often solutions were fully correct. Some candidates had their calculators in degree mode rather than radian mode and gave their answer in degrees. Rereading the question may have prevented this error. Other candidates would have done better if they had taken notice of the word "exact" in the question. Decimal answers were not accepted. Most candidates earned at least one of the two marks available.

Answers: **(i)** $x = 1 - \frac{\cot\theta}{2} - \frac{\operatorname{cosec}\theta}{2}$ **(iii)** $\frac{\pi}{3}$ radians

Question 10

(a) The many candidates who carefully and correctly formed **p** + 2**q** and equated to the given expression, answered this part of the question well.

(i) Some candidates made sign errors. Other candidates misinterpreted the given expression as $(\alpha + \beta)(\mathbf{i} - 20\mathbf{j})$. Few candidates used the convention of underlining vectors to make them distinct from the scalars. Perhaps some candidates would have reduced the number of errors they made if they had used this convention.

(ii) Candidates answered this part well. Most realised they needed to find the magnitude of 15**i** – 20**j** and divide by it. A small number of candidates only found the magnitude or only found 15**i** – 20**j**.

(b) There were a good number of fully correct solutions seen. Some candidates did not collect the components in **a.** Again, it may be that, if candidates use the convention of underlining vectors when writing them in lower case font, they realise the need to collect the components to complete the answer to the question. Several candidates misread or misinterpreted the given ratio. Candidates who did this were less likely to have marked values on the diagram. This may have helped reduce the interpretation errors made.

(c) Successful candidates used one of two methods to answer this part. Either they formed a correct proportional relationship using ratios of scalars and solved, or they introduced a second scalar and formed a pair of equations to solve in that scalar. A very good number of candidates scored full marks here. A small number omitted to discard the negative solution to the equation. A few candidates stated that parallel vectors have equal magnitude or equated the magnitudes and attempted to solve. Candidates who attempted to use the magnitude in some way were not successful.

Answers: **(a)(i)** α = 2, β = 13 **(ii)** $\frac{15\mathbf{i} - 20\mathbf{j}}{25}$ **(b)** $(1-\lambda)\mathbf{a} + \lambda\mathbf{b}$ **(c)** $\mu = 3$

Question 11

(i) This question was very well answered. A few candidates forgot that when taking a square root they should also have considered the negative solution to check if it was an appropriate solution. Rereading the question may have helped these candidates, as they are clearly asked for the stationary points rather than a stationary point. Most candidates used the quotient rule with only a few using the product rule. Some candidates evaluated y as $\frac{-1}{-1+1}$ when $x = -1$ and therefore discarded that point. Some candidates mismatched x- and y-coordinates or made careless sign errors when finding the y-coordinates. A few candidates found the x-coordinates only.

(ii) Most candidates applied the chain rule for differentiating a function of a function and did so correctly, Few candidates were able to manipulate the algebra successfully to the required form. Some of these candidates did see that the key to the solution was to factor out $(x^2 + 1)$ and cancel. Some candidates did manage to divide out at a later stage in their solution, but this was far less straightforward. Some good solutions were spoiled by sign errors. More attempts using the product rule were seen in this part.

Several candidates made no attempt to find the nature of the stationary points. Again, rereading the question may have prevented this omission. The candidates that did attempt to determine the nature of the stationary points all used the second derivative test. These candidates were generally successful. A small number made an incorrect determination and some candidates did not state the test fully or clearly enough to be credited.

Answers: **(i)** $\left(1,\ \frac{1}{2}\right)$, $\left(-1,\ -\frac{1}{2}\right)$ **(ii)** $\frac{d^2y}{dx^2} = \frac{2x^3 - 6x}{(x^2+1)^3}$, maximum at $x = 1$, minimum at $x = -1$

Question 12

(i) Several candidates verified that $t = 2$ resulted in $v = 0$ by substituting the value 2 into the expression for v and showing that it came to 0. This was partially correct, but not sufficient for full credit as these candidates had not shown this was the first point at which $v = 0$. These candidates would have done better if they had realised that, when asked to show that something is true, they should apply a method which results in that value, rather than simply using the given value to verify the given result. In this case, that required solving the quadratic expression for $v = 0$ and arriving at the two solutions $t = 2$ and $t = 5$, resulting in the first point of instantaneous rest being when $t = 2$, as given.

(ii) Virtually all candidates found $\frac{dv}{dt}$ correctly and were able to answer this part correctly. Candidates all showed their method of solution. No candidate seemed over-reliant on their calculator.

(iii) Similarly, virtually all candidates integrated the expression for v with respect to t and did so correctly to find an expression for the displacement. Some candidates omitted to calculate or state that c was equal to 0. A very small number of candidates would improve if they took a little more care with their variables. The terms in t were usually integrated with respect to t but the constant was, on occasion, integrated as $90x$ rather than $90t$.

(iv) Many candidates had a good understanding of the relationship between distance and displacement. Candidates scored well in this part, with a good proportion scoring full marks.

(a) Almost all candidates answered this part correctly. The few errors seen were giving the answer as $78 + c$, or for calculating s when $t = 1$ and when $t = 2$ and summing the results.

(b) A good number of fully correct answers were given. Among those who were not fully correct, a good number were partially correct, giving an answer of 67.5 m.

Answers: **(ii)** $a = 0$ ms^{-2} **(iii)** $(s =)\frac{9t^3}{3} - \frac{63t^2}{2} + 90t$ **(iv)(a)** 78 m **(b)** 88.5 m