

Cambridge IGCSE[™]

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

1364711255

CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/51

Paper 5 Investigation (Core)

October/November 2021

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [].

This document has 8 pages. Any blank pages are indicated.

Answer all the questions.

ADDING SQUARE NUMBERS

This investigation looks at adding two or more square numbers to make another square number. In this investigation all numbers are positive integers.

1 Complete the list of the first six square numbers.

$$1^2 = 1$$
 $2^2 = \dots$ $3^2 = 9$ $4^2 = \dots$ $5^2 = \dots$ $6^2 = 36$ [1]

- 2 (a) Work out
 - (i) 9²,[1]
 - (ii) 40^2[1]
 - **(b)** Show that $9^2 + 40^2 = 41^2$.

[2]

3

When $a^2 + b^2 = c^2$ then (a, b, c) is a 3-square set. a, b and c are positive integers.

Example

In **Question 2(b)**,
$$a = 9$$
, $b = 40$ and $c = 41$. $9^2 + 40^2 = 41^2$, so $(9, 40, 41)$ is a 3-square set.

When
$$a^2 + b^2 = c^2$$
 then $c = \sqrt{a^2 + b^2}$.

Use this formula and any patterns you notice to complete the table on the next page for 3-square sets.

а	b	С
3	4	5
5	12	13
7	24	25
9	40	41
11	60	
13	84	85
	112	113
	144	
19		181
21		221
25	312	313

[6]

4 When $a^2 + b^2 + c^2 = d^2$ then (a, b, c, d) is a 4-square set. It is possible to make a 4-square set using two rows in the table.

Example From the table row two row six

$$5^2 + 12^2 = 13^2$$
$$13^2 + 84^2 = 85^2$$

Replace 13^2 in the second equation with $5^2 + 12^2$ from the first equation: $5^2 + 12^2 + 84^2 = 85^2$.

So (5, 12, 84, 85) is a 4-square set.

Use the same method with rows from the table to find two more 4-square sets.

5 (a) Show that (6, 12, 12, 18) is a 4-square set.

[2]

(b)
$$k$$
 is any positive integer greater than 1.
If (ka, kb, kc, kd) is a 4-square set, then $(ka)^2 + (kb)^2 + (kc)^2 = (kd)^2$.

Show that (a, b, c, d) must also be a 4-square set.

[2]

(c)	The	numbers in the 4-square set (6, 12, 12, 18) have common factors.
	(i)	Find a common factor of 6, 12, 12 and 18 that is greater than 1.
		[1]
	(ii)	Use $(6, 12, 12, 18)$ and part (b) to find a 4-square set where a, b, c and d do not have a common factor greater than 1.
		(, ,, ,) [2]

6 Here is another method for finding a 4-square set (a, b, c, a)	6	Here is	another	method	for	finding	a 4-sc	luare	set ((a.	<i>b</i> .	c.	d)
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Choose two positive integers a and b with a less than b.

Then $c = \frac{a^2 + b^2 - 1}{2}$ and $d = \frac{a^2 + b^2 + 1}{2}$ make the 4-square set (a, b, c, d).

- (a) Use this to find a 4-square set when
 - (i) a = 2 and b = 3,

(ii) a = 2 and d = 43.

(2,, 43) [3]

(b) (i) Use your answers to **part (a)** and any patterns you notice to complete the table for 4-square sets that start with 2.

а	b	С	d
2	3		
2	5	14	15
2	7	26	27
2			43
2			

[3]

(ii) Write down an equation connecting c and d.

.....[1]

	When a and b are both even then 4-square set.	$c = \frac{a^2 + b^2 - 1}{2}$	and	$d = \frac{a^2 + b^2 + 1}{2}$	do not give a
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Give an example to show this.

[2]

(d) When a and b are both odd there are no 4-square sets.

In a 4-square set, d = 23.

(i) Show that $a^2 + b^2 = 45$.

[1]

(ii) Find a 4-square set when d = 23.

(....., 23) [2]

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