## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
You may use a pencil for any diagrams or graphs.

Answer all the questions.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.

At the end of the examination, fasten all your work securely together.
The total of marks for this paper is 24.

## UNIT FRACTIONS

A document called the Rhind Papyrus is nearly 4000 years old. It shows how the ancient Egyptians were able to break any fraction down into unit fractions. A unit fraction has a numerator of 1.

1 Add these unit fractions showing your working.
(a) $\frac{1}{8}+\frac{1}{6}$
(b) $\frac{1}{6}+\frac{1}{4}$

2 (a) Show that
(i) $\frac{1}{2}=\frac{1}{3}+\frac{1}{6}$,
(ii) $\frac{1}{3}=\frac{1}{4}+\frac{1}{12}$,
(iii) $\frac{1}{4}=\frac{1}{5}+\frac{1}{20}$.
(b) Break down these fractions into the sum of two unit fractions by continuing the pattern of question 2 (a).
$\frac{1}{5}=$
$\frac{1}{6}=$
$\frac{1}{7}=$
(c) Break down $\frac{1}{99}$ into the sum of two different unit fractions.

3 (a) Multiply each side of the result in question 2 (a) (ii) by 2.
Show that this simplifies to give $\frac{2}{3}=\frac{1}{2}+\frac{1}{6}$.
(b) Use the same method with the results in question 2 (b) to break down the following into the sum of two unit fractions.
(i) $\frac{2}{5}$
(ii) $\frac{2}{7}$
(c) Similarly, break down $\frac{10}{99}$ into the sum of two unit fractions.

4 On another Egyptian document there is a formula for breaking down any fraction of the form $\frac{a}{n}$ into the sum of two unit fractions.

The formula shown for doing this is
$\frac{a}{n}=\frac{a}{x y}=\frac{1}{k x}+\frac{1}{k y}$ where $k=\frac{x+y}{a}$ and $n=x \times y$.

An example To break down $\frac{4}{15}$ follow the steps below.

Step $1 \quad a=4$ and $n=15$.
Step 215 can be written as $3 \times 5$ so take $x=3$ and $y=5$.
Step $3 k=\frac{x+y}{a}=\frac{3+5}{4}=2$.

Step $4 \quad \frac{4}{15}=\frac{1}{2 \times 3}+\frac{1}{2 \times 5}=\frac{1}{6}+\frac{1}{10}$.
(a) Add $\frac{1}{6}+\frac{1}{10}$ and check that you get $\frac{4}{15}$.
(b) Use the method shown above to break down the following into two unit fractions.
(i) $\frac{4}{27}$
(ii) $\frac{7}{33}$
(c) In the example, $\frac{4}{15}$ can be broken down in a different way by choosing other values for $x$ and $y$ in Step 2.

What are these other values?
What is then the corresponding value of $k$ ?
Write down how $\frac{4}{15}$ is broken down in this case.
(d) Find all the different ways in which $\frac{3}{20}$ can be broken down into two unit fractions.
(e) 1 is equal to $\frac{1}{2}+\frac{1}{2}$.

Use this fact to break down 1 into the sum of three different unit fractions.
(f) Use your answer to question 4 (e) and any results already found to write 1 as the sum of four different unit fractions.

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