

**MARK SCHEME for the May/June 2015 series**

**9795 FURTHER MATHEMATICS**

**9795/02**

Paper 2 (Further Application of Mathematics),  
maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE<sup>®</sup>, Cambridge International A and AS Level components and some Cambridge O Level components.

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<p>1 (i)</p> <p>(ii)</p>	$2\mu - 50 = 0 \Rightarrow \mu = 25$ $44 + 25\sigma^2 = 144 \Rightarrow \sigma^2 = 4$ $X - Y \sim N(15, 15)$ $z = \frac{10 - 15}{\sqrt{15}} = -1.291$ $P(X - Y > 10) = \mathbf{0.902}$	<p>M1A1</p> <p>M1A1</p> <p>[4]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p><math>44 + 5\sigma^2</math> or <math>22 + 5\sigma^2</math>  <math>\rightarrow 20</math> or <math>24.4</math>: M1A0</p> <p><math>N(\mu - 10, 11 + \sigma^2)</math>  Standardise, including <math>\sqrt{\sigma^2}</math>  Allow <math>+1.29(1)</math></p> <p><math>[N(15, 31) \rightarrow 0.898 \rightarrow 0.815</math>:  M1M1A0A0]</p>																		
<p>2 (i)</p> <p>(ii)</p>	$\bar{x} = \frac{114}{20} = 5.7, \quad s^2 = \frac{2.382}{19} = 0.1254$ $t_{19} = 2.5395$ $98\% \text{ c.l.: } 5.7 \pm \left( 2.5395 \times \frac{\sqrt{0.1254}}{\sqrt{20}} \right)$ <p>98% C.I. is <b>(5.499, 5.901)</b></p> <p>5.5 is (just) within the confidence interval. Some evidence to suggest that the average pH is 5.5 in villages where rhododendrons grow well.</p>	<p>B1B1</p> <p>B1</p> <p>M1</p> <p>A1A1</p> <p>[6]</p> <p>B1FT</p> <p>B1FT</p> <p>[2]</p>	<p>Normal: B2B0M1A0</p> <p>Relate to CI  Conclusion  FT on their confidence interval</p>																		
<p>3 (i)</p> <p>(ii)</p>	$G'(t) = \frac{1}{81} \times 4 \left( t + \frac{2}{t} \right)^3 \left( 1 - \frac{2}{t^2} \right)$ $E(X) = G'(1) = \frac{4}{81} \times 27 \times (-1) = -\frac{4}{3}$ $G''(t) = \frac{4}{27} \left( t + \frac{2}{t} \right)^2 \left( 1 - \frac{2}{t^2} \right)^2 + \frac{4}{81} \left( t + \frac{2}{t} \right)^3 \times \frac{4}{t^3}$ $G''(1) = \frac{4}{27} \times 9 \times 1 + \frac{4}{81} \times 27 \times 4 = \frac{20}{3}$ $\text{Var}(X) = \frac{20}{3} + \left( -\frac{4}{3} \right)^2 - \frac{16}{9} = \frac{32}{9}$ <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;"><math>x</math></td> <td style="width: 10%; text-align: center;">-4</td> <td style="width: 10%; text-align: center;">-2</td> <td style="width: 10%; text-align: center;">0</td> <td style="width: 10%; text-align: center;">2</td> <td style="width: 10%; text-align: center;">4</td> </tr> <tr> <td><math>y = \frac{1}{2}(x+4)</math></td> <td style="text-align: center;">0</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">3</td> <td style="text-align: center;">4</td> </tr> <tr> <td><math>P(X=x)=P(Y=y)</math></td> <td style="text-align: center;"><math>\frac{16}{81}</math></td> <td style="text-align: center;"><math>\frac{32}{81}</math></td> <td style="text-align: center;"><math>\frac{24}{81}</math></td> <td style="text-align: center;"><math>\frac{8}{81}</math></td> <td style="text-align: center;"><math>\frac{1}{81}</math></td> </tr> </table> <p>Recognising as terms of the expansion of <math>\left( \frac{2}{3} + \frac{1}{3} \right)^4</math></p> <p>State <math>n = 4</math> and <math>p = \frac{1}{3}</math></p>	$x$	-4	-2	0	2	4	$y = \frac{1}{2}(x+4)$	0	1	2	3	4	$P(X=x)=P(Y=y)$	$\frac{16}{81}$	$\frac{32}{81}$	$\frac{24}{81}$	$\frac{8}{81}$	$\frac{1}{81}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1A1</p> <p>[5]</p> <p>B1</p> <p>B1</p> <p>B1B1</p> <p>[4]</p>	<p>For information:</p> $G(t) = \frac{1}{81} \left( t^4 + 8t^2 + 24 + \frac{32}{t^2} + \frac{16}{t^4} \right)$ $G'(t) = \frac{1}{81} \left( 4t^3 + 16t - \frac{64}{t^3} - \frac{64}{t^5} \right)$ $G''(t) = \frac{1}{81} \left( 12t^2 + 16 + \frac{192}{t^4} + \frac{320}{t^6} \right)$ <p><math>x</math> and probabilities  <math>y</math>  Or:  <math>G_{X+4}(t) = t^4 G_X(t)</math>  <math>G_{\frac{1}{2}(X+4)}(t) = G_{X+4}(\sqrt{t})</math></p> <p>Independent of method BUT max 3 if binomial not shown</p>
$x$	-4	-2	0	2	4																
$y = \frac{1}{2}(x+4)$	0	1	2	3	4																
$P(X=x)=P(Y=y)$	$\frac{16}{81}$	$\frac{32}{81}$	$\frac{24}{81}$	$\frac{8}{81}$	$\frac{1}{81}$																

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<p>4 (i) (a)</p> <p>(b)</p> <p>(ii) (a)</p> <p>(b)</p> <p>(c)</p>	$M(t) = \sum \frac{\lambda^r}{r!} e^{-\lambda} \cdot e^{tr} = e^{-\lambda} \sum \frac{(\lambda e^t)^r}{r!}$ $= e^{-\lambda} \cdot e^{\lambda e^t} = e^{\lambda(e^t - 1)}$ $M_Z(t) = M_X(t) \cdot M_Y(t) = e^{\mu(e^t - 1)} \cdot e^{v(e^t - 1)}$ $= e^{(\mu+v)(e^t - 1)} \Rightarrow Z \sim \text{Po}(\mu + v)$ <p>From tables <math>2 + k = 3.1 \Rightarrow k = 1.101 = \mathbf{1.10}</math> (3sf)</p> $P(3) = e^{-1.1} \times \frac{1.1^3}{3!} = \mathbf{0.0740}$ <p>Using mean of 3.1: <math>P(\leq 5) - P(\leq 1)</math>  <math>= 0.9057 - 0.1847 = 0.7210 = \mathbf{0.721}</math> (3 sf)</p>	<p>M1</p> <p>M1A1</p> <p>[3]</p> <p>M1</p> <p>A1</p> <p>[2]</p> <p>M1A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p> <p>[2]</p>	<p>Allow via expansion</p> <p>Needs to recognise series for M1 PGF quoted: M0M0A0</p> <p>Multiply two MGFs</p> <p>Needs one intermediate step</p> <p>Or <math>e^{-(2+k)} = 0.045</math>  <math>\Rightarrow 2 + k = \ln 22.22</math>  <math>\Rightarrow k = 1.101 = 1.10</math> (3sf)</p> <p>Or <math>P(\leq 3) - P(\leq 2) = 0.9743 - 0.9004</math>  Answer in range [0.0738, 0.0740]  <math>\lambda = k</math> needed for M1</p> <p>Or from series <math>\pm 1</math> term</p>
<p>5</p>	$\frac{60.5 - \mu}{\sigma} = 2.083, \quad \frac{39.5 - \mu}{\sigma} = 1.417$ $\mu = \mathbf{48.0}, \sigma = \mathbf{6}$ $\mu = np, \sigma = \sqrt{npq}$ $1 - p = 36 \div 48 = \frac{3}{4}$ $\Rightarrow \mathbf{p = 0.25, n = 192}$	<p>B1M1</p> <p>A1A1</p> <p>M1A1</p> <p>B1B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[11]</p>	<p><math>z</math> correct to 3 sf,</p> <p>Allow 3 sf, can be implied</p> <p><math>p \in [0.248, 0.250]</math>  192 or 193, must be integer</p>
<p>6 (i)</p>	$f_u(u) = \frac{1}{40} \quad U = g^{-1}(V) = \frac{40V}{V-40}$ $f_v(v) = f_u(g^{-1}(v)) \times  g^{-1}'(v) $ $= \frac{1}{40} \times \left  \frac{-1600}{(v-40)^2} \right  = \frac{40}{(v-40)^2} \text{ (AG)}$ <p>Or: <math>F_u(u) = \frac{u-80}{40}; \quad U = \frac{40V}{V-40}</math></p> $P(V < v) = P\left(U > \frac{40V}{V-40}\right) = 1 - P\left(U < \frac{40V}{V-40}\right)$ $= 2 - \frac{40}{v-40} \text{ so } f_v(v) = \frac{40}{(v-40)^2} \text{ (AG)}$	<p>B1</p> <p>M1</p> <p>M1A1</p> <p>A1</p> <p>[5]</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1A1</p> <p>[5]</p>	<p>PDF of <math>u</math></p> <p><math>U</math> in terms of <math>V</math></p> <p>Formula; mod sign needed for A1</p> <p>Mod sign needed for A1</p> <p>CDF of <math>U</math></p> <p><math>U</math> in terms of <math>V</math></p> <p>Turn <math>F_U(u)</math> into <math>F_V(v)</math>, allow no 1-</p> <p>Correct <math>F_V(v)</math>; correctly obtain AG</p>

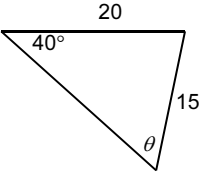
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<p><b>(ii) (a)</b></p>	$F(v) = 2 - \frac{40}{(v-40)} \quad 60 \leq v \leq 80$ $2 - \frac{40}{v-40} = \frac{1}{2} \Rightarrow v = \frac{200}{3} \quad (\text{OE})$	<p>M1 A1 [2]</p>	<p>(Or by integration of pdf from 60 to median = 0.5; M1 needs limits or c) Allow from <math>1 - F(v)</math></p>
<p><b>(b)</b></p>	$E(V) = \int_{60}^{80} \frac{40v}{(v-40)^2} dv = \int_{60}^{80} \frac{40}{v-40} + \frac{1600}{(v-40)^2} dv$ $= \left[ 40 \ln v-40  - \frac{1600}{v-40} \right]_{60}^{80}$ $= 40 \ln 2 + 40 \quad (= 67.7)$	<p>M1 M1A1 M1A1 A1 [6]</p>	<p>Or by <math>x = v - 40</math>: <math>40 \left[ \ln x - \frac{40}{x} \right]_{20}^{40}</math></p>
<p><b>7 (i)</b></p>	$x^2 + 9a^2 = (4a - x)^2 \Rightarrow \dots \Rightarrow x = \frac{7}{8}a \quad (\text{AG})$	<p>M1A1 [2]</p>	
<p><b>(ii)</b></p>	$T \cos \theta = mg$ $T + T \sin \theta = \frac{mv^2}{r}$ $\cos \theta = \frac{24}{25}, \quad \sin \theta = \frac{7}{25} \quad \text{or} \quad \tan \theta = \frac{7}{24}$ $\text{Solve to obtain } v = \sqrt{\frac{7ag}{6}}$	<p>B1 M1*A1 B1 dep*M1 A1 [6]</p>	<p>M1 needs two forces One correct, may be implied Solve</p>
<p><b>8 (i)</b></p>	<p>Hooke's law: <math>T = \frac{8x}{0.4} = 20x</math></p> <p>Newton II: <math>\frac{1}{5}\ddot{x} = -20x</math></p> <p><math>\Rightarrow \ddot{x} = -100x</math> which is SHM</p> <p>Period = <math>\frac{2\pi}{10} = \frac{1}{5}\pi</math> seconds</p>	<p>B1 M1 A1 A1 [4]</p>	<p>Can be specific <math>x</math> Needs general <math>x</math>, – sign A.e. exact f.</p>
<p><b>(ii)</b></p>	$-0.1 = 0.2 \cos 10t$ $\Rightarrow 10t = \frac{2\pi}{3} \Rightarrow t = \frac{1}{15}\pi \text{ seconds}$	<p>M1A1 M1A1 [4]</p>	<p>Or: Obtain <math>\frac{1}{60}\pi</math> Add quarter period to get <math>\frac{1}{15}\pi</math></p>

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<b>9 (i)</b>	<p>Gain in KE = <math>\frac{1}{2} \times 800 \times (25^2 - 10^2) = 210\,000\text{ J}</math>            Loss in PE = <math>800 \times 10 \times 400 \sin 2^\circ = 111\,678\text{ J}</math>            Work done = <math>210\,000 - 111\,678 = \mathbf{98\,322\text{ J}}</math>            (= 98.3 kJ)</p>	<p>M1 A1 A1 [3]</p>	<p><math>\frac{1}{2}mv^2 - \frac{1}{2}mu^2 - mg \times 400 \sin 2^\circ</math> Sign wrong: M1A0</p>
<b>(ii)</b>	<p><math>a = \frac{25^2 - 10^2}{800} = 0.65625</math> or <math>\frac{21}{32}</math>  <math>v^2 = 10^2 + 400 \times 0.65625 \Rightarrow v = 19.0394</math>  <math>F + 800 \times 10 \times \sin 2^\circ = 800 \times 0.65625 \Rightarrow F = 245.8</math>  <math>P = Fv = 4680 \Rightarrow</math> Power is <b>4.68 kW</b>.</p>	<p>M1A1 A1 M1A1 A1 [6]</p>	<p>Allow in part (i) only if used in part (ii) 3 terms needed for M1</p>
<b>10 (i)</b>	<p><math>-(mg + mkv^2) = m \frac{dv}{dt}</math>  <math>-\int_0^T dt = \int_u^0 \frac{1}{g + kv^2} dv</math>  <math>= \frac{1}{k} \int_u^0 \frac{1}{\frac{g}{k} + v^2} dv</math>  <math>T = \frac{1}{k} \left[ \sqrt{\frac{k}{g}} \tan^{-1} \sqrt{\frac{k}{g}} v \right]_0^u</math>  <math>= \frac{1}{\sqrt{gk}} \tan^{-1} \sqrt{\frac{k}{g}} u</math></p>	<p>B1 M1 M1 A1 A1 [5]</p>	<p>Allow + only if <math>\downarrow</math> explicit Separate and insert integral signs  Or indefinite integral and find <math>c</math> Correct indefinite integral</p>
<b>(ii)</b>	<p><math>-(mg + mkv^2) = mv \frac{dv}{dx}</math>  <math>-\int_0^H dx = \frac{1}{2k} \int_u^0 \frac{2kv}{g + kv^2} dv</math>  <math>H = \frac{1}{2k} \left[ \ln  g + kv^2  \right]_0^u</math>  <math>= \frac{1}{2k} \ln \left( \frac{g + ku^2}{g} \right)</math></p>	<p>B1 M1 M1 A1 A1 [5]</p>	<p>Allow + only if <math>\downarrow</math> explicit  Or indefinite integral and find <math>c</math> Correct indefinite integral</p>

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<p><b>11 (i)</b></p> <p>Right angled triangle with angle</p> $\tan^{-1} \frac{4}{3} - 50^\circ = 3.13^\circ$ <p>Shortest distance = <math>5 \sin 3.13^\circ = \mathbf{0.273 \text{ km}}</math></p> <p><b>(ii) (a)</b> A correct velocity triangle.</p> $\frac{\sin \theta}{20} = \frac{\sin 40^\circ}{15} \Rightarrow \theta = 58.986\dots^\circ$ <p>Bearing is <math>\mathbf{008.99^\circ}</math></p>  <p><b>(b)</b></p> $\frac{v}{\sin 81.013\dots^\circ} = \frac{15}{\sin 40^\circ} \Rightarrow v = 23.049$ $\text{Time} = \frac{5}{23.049} \times 60 = \mathbf{13.0 \text{ minutes.}}$	<p>B1</p> <p>M1A1 [3]</p> <p>B1</p> <p>M1A1</p> <p>A1 [4]</p> <p>M1A1</p> <p>M1A1 [4]</p>	<p><math>(20t - 5 \sin 50)^\circ + (5 \cos 50 - 15t)^2</math> <math>\Rightarrow t = 0.1997</math></p> <p>(Accept <math>9^\circ</math> or <math>8.99^\circ</math>)</p> <p>(Accept 13 minutes.)</p>
<p><b>12 (i)</b></p> <p>Taking axes along and perpendicular to plane:</p> $\dot{x} = u \cos \theta - gt \sin \alpha; \quad \dot{x} = 0 \Rightarrow t = \frac{u \cos \theta}{g \sin \alpha}$ $y = ut \sin \theta - \frac{1}{2}gt^2 \cos \alpha; \quad y = 0 \Rightarrow t = \frac{2u \sin \theta}{g \cos \alpha}$ $\frac{u \cos \theta}{g \sin \alpha} = \frac{2u \sin \theta}{g \cos \alpha} \Rightarrow 2 \tan \alpha \tan \theta = 1 \quad (\text{AG})$ <p><b>(ii) (a)</b></p> $x = u \cos \theta \cdot \frac{u \cos \theta}{g \sin \alpha} - \frac{1}{2}g \sin \alpha \cdot \frac{u^2 \cos^2 \theta}{g^2 \sin^2 \alpha}$ $\Rightarrow x \sin \alpha = \frac{u^2}{2g} \cos^2 \theta$ <p>But <math>\cos^2 \theta = \frac{1}{1 + \tan^2 \theta} = \frac{1}{1 + \frac{1}{4 \tan^2 \alpha}} = \frac{4 \tan^2 \alpha}{1 + 4 \tan^2 \alpha}</math></p> $\Rightarrow x \sin \alpha = \frac{2u^2 \tan^2 \alpha}{g(1 + 4 \tan^2 \alpha)} \quad (\text{AG})$ <p><b>(b)</b></p> <p>Time of flight = <math>\frac{u \cos \theta}{g \sin \alpha} = \frac{u}{g \sin \alpha} \sqrt{\frac{4 \tan^2 \alpha}{1 + 4 \tan^2 \alpha}}</math></p> $= \frac{2u}{g \cos \alpha \sqrt{1 + 4 \tan^2 \alpha}} = \frac{2u \sec \alpha}{g \sqrt{1 + 4 \tan^2 \alpha}} \quad (\text{AG})$ <p>(N.B. Other orders of doing this may be seen.)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1A1 [6]</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1 [5]</p> <p>M1</p> <p>M1A1 [3]</p>	<p>Equation for <math>\dot{x}</math>, equated to 0 Find (eliminate) <math>t</math></p> <p>Equation for <math>y</math>, equated to 0 Find (eliminate) <math>t</math></p> <p>Equate and simplify; get AG</p> <p>Equation for <math>x</math>, substitute <math>t</math></p> <p>Correct expression for <math>x \sin \alpha</math></p> <p><math>\cos \theta</math> in terms of <math>\tan \alpha</math></p> <p>Obtain given answer</p> <p>Find <math>t</math> in terms of <math>\tan \alpha</math></p> <p>Simplify to given answer</p>

Alternative method for 12:

<p><b>12 (i)</b></p>	<p>Taking axes horizontally and vertically, where <math>\beta = \alpha + \theta</math></p> <p><math>y = x \tan \beta - \frac{gx^2}{2u^2}(1 + \tan^2 \beta)</math> and <math>y = \tan \alpha</math></p> <p>(on landing)</p> <p><math>\Rightarrow \tan \alpha = \tan \beta - \frac{gx}{2u^2}(1 + \tan^2 \beta)</math> (I)</p> <p><math>\left(\frac{dy}{dx} = \right) \tan \beta - \frac{gx}{u^2}(1 + \tan^2 \beta) = \frac{-1}{\tan \alpha}</math> (II)</p> <p>(I) &amp; (II) <math>\Rightarrow \tan \beta + \frac{1}{\tan \alpha} = 2(\tan \beta - \tan \alpha)</math></p> <p><math>\Rightarrow \tan \beta = 2 \tan \alpha + \frac{1}{\tan \alpha}</math> (III)</p> <p><math>\Rightarrow \frac{\tan \alpha + \tan \theta}{1 - \tan \alpha \tan \theta} = \frac{2 \tan^2 \alpha + 1}{\tan \alpha} \Rightarrow \dots</math></p> <p><math>\Rightarrow 2 \tan \alpha \tan \theta = 1</math> (AG)</p>	<p>M1 A1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p><math>\left( = \frac{gx}{u^2}(1 + \tan^2 \beta) \right)</math></p>
<p><b>(ii) (a)</b></p>	<p>(I) &amp; (III) <math>\Rightarrow \frac{gx^2}{2u^2}(1 + \tan^2 \beta) = \frac{2 \tan^2 \alpha + 1}{\tan \alpha} - \tan \alpha</math></p> <p><math>= \frac{\tan^2 \alpha + 1}{\tan \alpha}</math></p> <p><math>\Rightarrow x \tan \alpha = \frac{2u^2}{g} \left\{ \frac{1 + \tan^2 \alpha}{1 + \tan^2 \beta} \right\} = \frac{2u^2}{g} \left\{ \frac{1 + t^2}{1 + \left(\frac{2t^2+1}{t}\right)^2} \right\}</math>,</p> <p>where <math>t = \tan \alpha</math>. (Required height is <math>x \tan \alpha</math>.)</p> <p><math>\Rightarrow \dots \Rightarrow x \tan \alpha = \frac{2u^2 \tan^2 \alpha}{g(1 + 4 \tan^2 \alpha)}</math> (AG)</p>	<p>M1A1</p> <p>M1A1</p> <p>A1</p> <p>[5]</p>	
<p><b>(b)</b></p>	<p><math>T = \frac{x}{u \cos \beta} = \frac{2u^2 t^2}{g(1 + 4t^2)} \cdot \frac{1}{t} \cdot \frac{1}{u \cos \beta}</math></p> <p><math>\Rightarrow T^2 = \frac{4u^2 t^2 (1 + \tan^2 \beta)}{g^2 (1 + 4t^2)^2} = \frac{4u^2 t^2 (1 + [2t + \frac{1}{t}]^2)}{g^2 (1 + 4t^2)^2} = \dots</math></p> <p><math>= \frac{4u^2}{g^2} \left( \frac{1 + t^2}{1 + 4t^2} \right) \Rightarrow T = \frac{2u \sec \alpha}{g \sqrt{1 + 4 \tan^2 \alpha}}</math> (AG)</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	