Cambridge International Examinations
Cambridge Pre-U Certificate

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FURTHER MATHEMATICS (PRINCIPAL) **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2017**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **5** printed pages and **3** blank pages.

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Section A: Probability (60 marks)

1 **(i)** Explain the meaning of the term '95% confidence interval'. [1]

(ii) The values of five independent observations of a normally distributed random variable are as follows.

35.2   38.2   39.7   41.6   43.9

Obtain a 95% confidence interval for the population mean. [5]

2 A discrete random variable X has the following probability distribution.

x	-1	2
$\mathrm{P}(X = x)$	$\frac{1}{3}$	$\frac{2}{3}$

(i) Write down the probability generating function of X. [2]

(ii) T is the sum of ten independent observations of X. Use the probability generating function of T to find

(a) $\mathrm{E}(T)$, [4]

(b) $\mathrm{P}(T = 8)$. [3]

3 In a random sample of 100 voters from a constituency, 32 said that they would support the Cyan Party.

(i) Find an approximate 99% confidence interval for the proportion of voters in the constituency who would support the Cyan Party. [3]

(ii) Using the given sample proportion, estimate the smallest size of sample needed for the width of a 99% confidence interval to be less than 0.04. [3]

4 A continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} a & -1 \leqslant x < 0, \\ a(1 - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Find the value of a. [3]

(ii) Find the cumulative distribution function of X. [5]

(iii) Determine whether the upper quartile is greater than or less than 0.25. [2]

5 The number of calls to a car breakdown service during any one hour of the day is modelled by the distribution $\mathrm{Po}(20)$.

(i) Find the probability that in a randomly chosen 12-minute period there are at least 7 calls to the service. [2]

(ii) Find the period of time, correct to the nearest second, for which the probability that no calls are made to the service is 0.6. [3]

(iii) Use a suitable approximation to find the probability that, in a randomly chosen 3-hour period, there are no more than 65 calls to the service. [5]

6 The random variable X has a uniform distribution on the interval $[-1, 1]$, so that its probability density function is given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{2} & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show from the definition of the moment generating function that the moment generating function of X is $\dfrac{\sinh t}{t}$. [4]

(ii) By using the series expansion of $\sinh t$, find the variance of X and the value of $\mathrm{E}(X^4)$. [5]

7 The total mass of a can of pears is the sum of three independent random variables: the mass of pears, the mass of juice, and the mass of the container. The mass in grams of pears in a can has the distribution $\mathrm{N}(300, 400)$. The mass in grams of juice has the distribution $\mathrm{N}(200, 60)$. The mass in grams of the container has the distribution $\mathrm{N}(70, 10)$.

(i) Find the probability that the total mass of a randomly chosen can is less than 530 g. [3]

(ii) Find the probability that the mass of the container of a randomly chosen can is more than one eighth of the total mass of the can. [7]

Section B: Mechanics starts on page 4.

Section B: Mechanics (60 marks)

8 A horizontal turntable rotates about a vertical axis. Starting from rest, it accelerates uniformly to an angular velocity of $8.4\,\text{rad}\,\text{s}^{-1}$ in $2\,\text{s}$.

(i) Find the angular acceleration of the turntable. [1]

(ii) A particle rests on the turntable at a distance of $0.15\,\text{m}$ from the axis. Find the radial and transverse components of the acceleration of the particle when the angular velocity is $1.2\,\text{rad}\,\text{s}^{-1}$. Find also the magnitude of the acceleration at this instant. [4]

9 A particle is projected from a point O on horizontal ground with speed $40\,\text{m}\,\text{s}^{-1}$ at an angle θ above the horizontal.

(i) Write down the equation of the trajectory, in terms of $\tan\theta$. [2]

(ii) The particle passes through a point whose horizontal and vertical distances from O are $72\,\text{m}$ and $y\,\text{m}$ respectively. By considering the equation of the trajectory as a quadratic equation in $\tan\theta$, or otherwise, find the greatest possible value of y. [5]

10 The engine of a lorry of mass $4000\,\text{kg}$ works at a constant rate of $75\,\text{kW}$. Resistance to motion is modelled by a constant resistive force. On a horizontal road the lorry travels at a constant speed of $24\,\text{m}\,\text{s}^{-1}$.

(i) Find the work done by the engine in travelling for 1 minute on the horizontal road. [2]

(ii) The lorry travels at a constant speed $v\,\text{m}\,\text{s}^{-1}$ up a slope of angle $\sin^{-1}0.05$ to the horizontal. Find the value of v. [6]

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A non-uniform rod AB of mass $1.6\,\text{kg}$ and length $1.25\,\text{m}$ has its centre of mass at G where $AG = 0.4\,\text{m}$. The rod rests on a rough horizontal table. A force $P\,\text{N}$ is applied at B, acting at an angle α above the horizontal, such that the rod is in equilibrium but about to rotate about A (see diagram).

(i) Assume that the rod is in contact with the table only at A. By taking moments about A, show that $P\sin\alpha = 5.12$. [3]

(ii) The coefficient of friction between the rod and the table is $\frac{6}{17}$. Show that $P \leqslant 6.4$. [6]

12 A particle of mass 0.6 kg is projected vertically upwards from horizontal ground, with initial speed $u\,\mathrm{m\,s^{-1}}$. The upwards velocity at any instant is $v\,\mathrm{m\,s^{-1}}$, and the displacement is x m. Air resistance is modelled by a force $0.024v^2$ N acting downwards.

(i) Show that v and x satisfy the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = -10 - 0.04v^2.$$ [2]

(ii) Find the value of u if the maximum height reached is 50 m. [7]

13 A fairground game consists of a small ball fixed to one end of a light inextensible string of length 0.6 m. The other end of the string is fixed to a point O. A small bell is fixed 0.6 m vertically above O. Initially the ball hangs vertically in equilibrium. The object of the game is to project the ball with an initial horizontal velocity $u\,\mathrm{m\,s^{-1}}$ so that it moves in a vertical circle and hits the bell.

(i) Find the smallest possible value of u for which the ball hits the bell. [7]

(ii) Given, instead, that the value of u is 5, find the angle made by the string with the upward vertical at the moment when the string becomes slack. [6]

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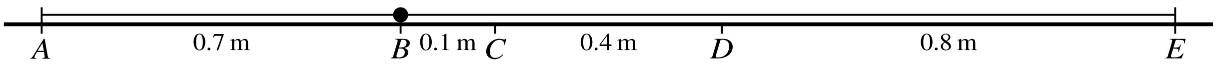

A particle of mass 0.05 kg is attached to two identical light elastic strings, each of natural length 1.2 m and modulus of elasticity 0.6 N. The other ends of the strings are attached to points A and E on a smooth horizontal table. The distance AE is 2 m and points B, C and D lie between A and E so that $AB = 0.7$ m, $BC = 0.1$ m, $CD = 0.4$ m and $DE = 0.8$ m (see diagram). Initially the particle is held at B and it is then released. In the subsequent motion the displacement of the particle from C, in the direction of A, is denoted by x m.

(i) Find the equation of motion for the particle when it is between B and C. [2]

(ii) Find the velocity of the particle when it is at C. [2]

(iii) Find the total time that elapses before the particle first returns to B. [5]

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