Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Application of Mathematics **For examination from 2020**

MARK SCHEME

Maximum Mark: 120

Specimen

This specimen paper has been updated for assessments from 2020. The specimen questions and mark schemes remain the same. The layout and wording of the front covers have been updated to reflect the new Cambridge International branding and to make instructions clearer for candidates.

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document has **10** pages. Blank pages are indicated.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.

The following abbreviations may be used in a mark scheme:

AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
aef	Any equivalent form
art	Answers rounding to
cwo	Correct working only (emphasising that there must be no incorrect working in the solution)
ft	Follow through from previous error is allowed
o.e.	Or equivalent
D	Dependent mark (dependent on an earlier mark in the scheme)

Question	Answer	Marks	Notes
1(a)	$G_X(t) = at\left(t+\frac{1}{t}\right)^3 = at(t^3 + 3t + 3t^{-1} + t^{-3}) = a(t^4 + 3t^2 + 3t^0 + t^{-2})$	M1	
	X takes the values 4, 2, 0, –2	A1	
	$G_X(1) = 1$ *or* sum of coefficients = 1	M1	
	$\Rightarrow a = \frac{1}{8}$	A1	
		4	
1(b)	$E(X) = G_X{}'(1) = a(4t^3 + 6t + 0 - 2t^{-3})\|_{t=1}$ Differentiate and evaluate at $t = 1$ **OR** by symmetry	M1	
	= 1 [**ft** $8a$]	A1	A1ft
		2	

Question	Answer	Marks	Notes
2(a)	$np = 100 \times \frac{1}{5} = 20$ and $npq = 20 \times \frac{4}{5} = 16$	B1	
	Standardisation $z = \frac{14.5 - 20}{4} = -1.375$	M1	
	$\Rightarrow P(\geqslant 15) = 0.915$ (within range [0.915, 0.916]) (**ft** on variance)	A1A1	A1ftA1
	Justified as $np = 20 > 5$ and $nq = 80 > 5$ **OR** n large, p not too far from ½	B1	
		5	
2(b)	mean = variance = 36 $\Rightarrow$ standard deviation = 6	B1	
	$z = 1.645$	B1	
	$\frac{\left(N+\frac{1}{2}\right)-36}{6} > 1.645$ (Allow working with equality, but must be $\left(N+\frac{1}{2}\right)$)	M1A1	
	$\Rightarrow N > 45.37 \quad \therefore$ least $N = 46$	A1	
		5	

Question	Answer	Marks	Notes
3(a)	$\bar{x} = 1.675$	B1	
	99% confidence limits are $1.675 \pm 2.576 \times \frac{0.1}{\sqrt{6}}$ (**ft** on wrong mean)	M1A1	M1A1ft
	99% confidence interval is (1.57, 1.78) art	A1	
		4	

Question	Answer	Marks	Notes
3(b)	$s_n = 0.09215$ **OR** $s_{n-1} = 0.1009$	B1	
	$v = 5 \Rightarrow t_5\,(0.99) = 4.032$	B1	
	99% confidence limits are $1.675 \pm 4.032 \times \dfrac{0.1009}{\sqrt{6}}$ **OR** $1.675 \pm 4.032 \times \dfrac{0.09215}{\sqrt{5}}$	M1A1	
	99% confidence interval is (1.51, 1.84) art	A1	
		5	
3(c)	Sensible comment referring to the fact that 1.8 is outside the 1st interval but inside 2nd. (**ft** on their confidence intervals)	B1	B1ft
		1	

Question	Answer	Marks	Notes
4(a)	$\mathrm{E}(a\overline{X} + b\overline{Y}) = \mu$	M1	
	$\mathrm{E}(a\overline{X} + b\overline{Y}) = a\mathrm{E}(\overline{X}) + b\mathrm{E}(\overline{Y})$	M1	
	$\Rightarrow a\mu + 3b\mu = \mu \Rightarrow a + 3b = 1$	A1	
		3	
4(b)	$\mathrm{Var}(a\overline{X} + b\overline{Y}) = a^2\mathrm{Var}(\overline{X}) + b^2\mathrm{Var}(\overline{Y}) = a^2\dfrac{\sigma^2}{n} + 4b^2\dfrac{\sigma^2}{n}$	M1	
	$= \dfrac{\sigma^2}{n}(a^2 + 4b^2) = \dfrac{\sigma^2}{n}(1 - 6b + 9b^2 + 4b^2) = \dfrac{\sigma^2}{n}(1 - 6b + 13b^2)$ **AG**	M1A1	
		3	
4(c)	$\dfrac{\mathrm{d}}{\mathrm{d}b}\mathrm{Var}(a\overline{X} + b\overline{Y}) = -6 + 26b = 0 \Rightarrow b = \dfrac{3}{13}$	M1A1	
	$\Rightarrow \mathrm{Var}_{\min}(a\overline{X} + b\overline{Y}) = \dfrac{\sigma^2}{n}\left(1 - 6 \times \dfrac{3}{13} + 13 \times \dfrac{9}{169}\right) = \dfrac{4\sigma^2}{13n}$	A1	
		3	

Question	Answer	Marks	Notes
5(a)	$M_X(t) = \int_0^\infty \mathrm{e}^{tx} k\mathrm{e}^{-kx}\,\mathrm{d}x$ (Limits required)	M1	
	$= k\int_0^\infty \mathrm{e}^{(t-k)x}\,\mathrm{d}x = k\int_0^\infty \mathrm{e}^{-(k-t)x}\,\mathrm{d}x$ (Limits not required)	M1	
	$= \dfrac{-k}{k-t}\left[\mathrm{e}^{-(k-t)x}\right]_0^\infty = \dfrac{k}{k-t}$ **AG**	A1	
		3	

Question	Answer	Marks	Notes
5(b)	$M_X{}'(t) = \frac{k}{(k-t)^2} \Rightarrow \mathrm{E}(X) = M_X{}'(0) = \frac{1}{k}$	M1A1	
	$M_X{}''(t) = \frac{2k}{(k-t)^3} \Rightarrow \mathrm{E}(X^2) = M_X{}''(0) = \frac{2}{k^2}$	M1A1	
	$\Rightarrow \mathrm{Var}(X) = \frac{2}{k^2} - \left(\frac{1}{k}\right)^2 = \frac{1}{k^2}$ (A1 **ft** if double sign error when differentiating twice, but CAO)	A1	
	OR **Alternatively:** $\mathrm{M}_X(t) = \left(1 - \frac{t}{k}\right)^{-1} = 1 + \frac{t}{k} + \frac{t^2}{k^2} + \ldots$	M1A1	
	$\mathrm{E}(X) = \frac{1}{k}$	A1	
	$E(X^2) = \frac{2}{k^2} \Rightarrow \mathrm{Var}(X) = \frac{2}{k^2} - \left(\frac{1}{k}\right)^2 = \frac{1}{k^2}$	M1A1	
	Available Marks	**5**	
5(c)	$\mathrm{E}(\mathrm{e}^{t[-(-X)]}) = \mathrm{E}(\mathrm{e}^{[-(-t)X]}) = \mathrm{M}_X(-t) = k(k+t)^{-1}$. Or equivalent	B1	
		1	
5(d)	$\mathrm{M}_{X_1 - X_2}(t) = \mathrm{M}_X(t) \times \mathrm{M}_{-X}(t) = k^2(k^2 - t^2)^{-1}$	M1	
	$= 1 + \frac{t^2}{k^2} + \frac{t^4}{k^4} + \ldots$ **OR** find $\mathrm{M}''(0)$	M1	
	$\Rightarrow \mathrm{E}(X_1 - X_2)^2 = 2! \times \text{coefficient of } t^2 = \frac{2}{k^2}$	A1	
		3	

Question	Answer	Marks	Notes
6(a)	**Above** x-axis between (0, 0) to (3, 0)	B1	
	Correct concavity. (Do not condone parabolas)	B1	
		2	
6(b)	$\mu = \frac{4}{27}\int_0^3 (3x^3 - x^4)\,\mathrm{d}x$ (Limits required)	M1	
	$= \frac{4}{27}\left[3\frac{x^4}{4} - \frac{x^5}{5}\right]_0^3 = 1.8$	A1A1	
	$\mathrm{f}'(x) = \frac{4}{27}(6x - 3x^2) = 0$	M1A1	
	$\Rightarrow x = 0, 2 \quad \therefore$ Mode = 2	A1	
		6	

Question	Answer	Marks	Notes
6(c)	Mean less than mode in **(b)** matches negative skew in sketch.	B1	
		1	
6(d)	$\left[\frac{4}{27}\left(x^3 - \frac{x^4}{4}\right)\right]_0^l = \frac{1}{4} \Rightarrow Q_1^{\ 4} - 4Q_1^{\ 3} + 6.75 = 0$ **AG**	B1	
	Use e.g. $Q_{1n+1} = Q_{1n} - \dfrac{Q_{1n}^{\ \ 4} - 4Q_{1n}^{\ \ 3} + 6.75}{4Q_{1n}^{\ \ 3} - 12Q_{1n}^{\ \ 2}}$ or decimal search etc.	M1A1	
	Convincingly obtain 1.37	A1	
		4	

Question	Answer	Marks	Notes
7	$\frac{1}{2} \times 20 \times 2^2 + 20 \times 8 \times 10$	M1	
	$= \frac{1}{2} \times 20 \times 6^2 + F \times 16$ All signs correct for A1	M1A1	
	Solve to get $F = 80$	A1	
		4	

Question	Answer	Marks	Notes
8(a)	$T \sin\theta = ml \sin \omega^2 \Rightarrow T = ml\,\omega^2$	M1A1	
		2	
8(b)	$T\dfrac{h}{l} = mg \Rightarrow T\dfrac{mgl}{h}$	B1	
	$\Rightarrow \omega^2 h = g.$	B1	
		2	
8(c)	Time is $\dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{h}{g}}$	M1A1	
		2	

Question	Answer	Marks	Notes
9	Resolve vertically $N_B = 2 \times 10$	M1	
	Take moments about e.g. intersection of normals: $20 \times 0.2 \cos 60 = F \times 0.4 \sin 60$	M1	
	Moments equation correct (if in equilibrium)	A1	
	$F = 5.77$, $N_B = 20$	A1	
	$F > \mu N_B$	M1	
	Correctly deduce not in equilibrium therefore rod does slip	A1	
		6	

Question	Answer	Marks	Notes
10(a)	Tractive force = Resistance at steady speed $\Rightarrow \frac{75}{100} = 10k \Rightarrow k = \frac{3}{4}$ **AG**	B1	
		1	
10(b)	$F = ma \Rightarrow \frac{75}{v} - \frac{3}{4}v = 90\frac{dv}{dt} \Rightarrow \frac{25}{v} - \frac{1}{4}v = 30\frac{dv}{dt}$ **AG** (3 terms required for M1)	M1A1	
		2	
10(c)	$\int_0^t dt = \int_3^7 \frac{120v}{100-v^2}dv$	M1	
	$t = -60\int_3^7 \frac{-2v}{100-v^2}dv = \left[-60\ln\lvert 100-v^2\rvert\right]_3^7$ (Limits not required)	M1A1	
	$= -60\ln 51 + 60\ln 91 = 60\ln\left(\frac{91}{51}\right)$ (= 34.7) seconds.	M1A1	
		5	

Question	Answer	Marks	Notes
11(a)	Let u denote speed of sphere Q before impact, v_1 and v_2 the speeds of spheres Q and P, respectively, after impact and α the angle between Q's initial direction of motion and the line of centres. After impact, if moving perpendicularly, Q moves perpendicular to line of centres and P moves along line of centres. (Stated or implied)	B1	
	Conservation of linear motion: $mu\cos\alpha = 0 + 3mv_2$ or $mu_x = 3mv$	M1A1	
	Newton's experimental law: $eu\cos\alpha = v_2$ or $eu_x = v$.	A1	
	$\therefore e = \frac{1}{3}$	A1	
		5	
11(b)	$v_1 = u\sin\alpha$ and $v_2 = \frac{1}{3}u\cos\alpha$ (both needed)	B1	
	Loss in kinetic energy is $\frac{1}{2}mu^2 - \frac{1}{2}mu^2\sin^2\alpha - \frac{1}{2}.3m\frac{u^2\cos^2\alpha}{9}$	M1A1	
	$= \frac{1}{12}mu^2$ (Or remaining kinetic energy is 5/6 of initial kinetic energy etc.)	A1	
	But $\cos^2\alpha + \sin^2\alpha = 1$ (used)	M1	
	$\Rightarrow \ldots \Rightarrow \sin^2\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 60°$	M1A1	
		7	

Question	Answer	Marks	Notes
12(a)	B1 for $x = 20\cos\alpha t$ B1 for $y = 20\sin\alpha t - 5t^2$	B1B1	
	$y = 20\sin\alpha . \frac{x}{20\cos\alpha} - 5\left(\frac{x}{20\cos\alpha}\right)^2 = x\tan\alpha - \frac{x^2}{80}(1+\tan^2\alpha)$ **AG**	M1A1	
		4	
12(b)	$x^2\tan^2\alpha - 80x\tan\alpha + x^2 + 80y = 0$ (Can be implied by what follows.)	B1	
	Real roots $\Rightarrow 6400x^2 - 4x^2(x^2 + 80y) \geqslant 0$	M1A1	
	$\Rightarrow 1600 - x^2 - 80y \geqslant 0 \Rightarrow y \leqslant 20 - \frac{x^2}{80}$, $(x \neq 0)$.	A1	
		4	
12(c)	$x^2 + 80x\tan 30 - 1600 = 0$ $x^2 + \frac{80}{\sqrt{3}}x - 1600 = 0$	B1	
	$x = -\frac{40}{\sqrt{3}} \pm \sqrt{\frac{1600}{3} + 1600}$	B1	
	$= -\frac{40}{\sqrt{3}} + 2\times\frac{40}{\sqrt{3}} = \frac{40}{\sqrt{3}}$ (ignore – at this stage)	B1	
	$\Rightarrow R = x \div \cos 30 = \frac{80}{3}$ (A0 if both solutions retained)	A1	
	OR **Alternative solution:** $x = R\cos 30°$ and $y = R\sin 30°$: $y = 20 - \frac{x^2}{80} \Rightarrow R\sin 30 = 20 - \frac{R^2(1-\sin^2 30)}{80}$	M1A1	
	$\therefore\ 0.75R^2 + 40R - 1600 = 0 \Rightarrow (0.5R + 40)(1.5R - 40) = 0$	M1	
	$\Rightarrow R = \frac{80}{3}$ (A0 if both solutions retained)	A1	
	Available marks	**4**	

Question	Answer	Marks	Notes
13(a)	Indication (e.g. from diagram) that each string has extension e given by $e = \sqrt{(3l)^2 + x^2} - l$	B1	
	Use of Hooke's Law, $T = \frac{\lambda e}{l}$, to get $T = \frac{6mg\left(\sqrt{9l^2 + x^2} - l\right)}{l}$ **AG**	B1	
		2	

Question	Answer	Marks	Notes
13(b)	Let θ be the angle between each string and line of motion of particle.	M1	
	$m\ddot{x} = -2T\cos\theta = -\frac{12mg}{l}\left(\sqrt{9l^2+x^2}-l\right)\times\frac{x}{\sqrt{9l^2+x^2}}$	A1A1	
	$\Rightarrow \ddot{x} = -\frac{12gx}{l}\left(1-\frac{l}{\sqrt{9l^2+x^2}}\right)$ **AG**	A1	
		4	
13(c)	$\therefore \ddot{x} \approx (-12g+4g)\frac{x}{l} = -\frac{8g}{l}x$ **AG**	M1A1	
		2	
13(d)	$v_{\max} = \omega a \Rightarrow \frac{gl}{200} = \frac{8g}{l}a^2 \Rightarrow a^2 = \frac{l^2}{1600} \Rightarrow a = \frac{1}{40}l$	M1A1	
	Hence time t given by $\frac{1}{80}l = a\sin\sqrt{\frac{8g}{l}}t \Rightarrow t = \frac{\pi}{6}\sqrt{\frac{l}{8g}}$ o.e.	M1A1	
		4	